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FAO Fisheries Biology Technical Paper No. 26
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FB/T26
MFS - Sampling

MANUAL OF SAMPLING METHODS FOR FISHERIES BIOLOGY

by

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FISHERIES DIVISION, BIOLOGY BRANCH
FOOD AND AGRICULTURE ORGANIZATION OF THE UNITED NATIONS
Rome, October 1962

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FAO FISHERIES BIOLOGY TECHNICAL PAPER (MFS)

Documents in this group are provisional editions of volumes or sections of FAO MANUALS IN FISHERIES SCIENCE, usually in the language of the original draft. They are given a limited distribution for comment by collaborators in this project, and for use of FAO headquarters and field staff and at training centres, courses and seminars. Revised versions are subsequently published by FAO in the official languages of the Organization.

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Expanded version of J.A. Gulland (1957) FAO Fish. bull., 10 (4): 157-81, revised on basis of author's experience at FAO/EPTA Training Centre on Methodology of Research on Mackerel (Rastrelliger), held at Bangkok, Thailand, 1958, and Training Course on Fish Population Dynamics at Lowestoft, England, 1957. This version, prepared at the request of the IPFC (Karachi meeting, 1960), is for use at seminars to be given by the author in Australia, October-November 1962, and at 2nd Training Course on Fish Stock Assessment to be held at Lowestoft in February-March, 1963. Comments on it, requests for additional copies, should be addressed to Chief, Biology Branch, Fisheries Division, FAO, Rome, Italy.

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1 INTRODUCTION AND GENERAL STATISTICS

1.1 Introduction

Most of the quantities involved in fishery research cannot be observed or measured throughout the whole population, e.g. it is virtually impossible to measure all the fish caught, still less all the fish in the sea. A section, or sample, of the whole population is therefore examined for the attributes concerned, e.g. percentage of mature fish, or average size. On the assumption that this sample is in some way representative of the whole population, an estimate can be made of the true value in the population. If the sampling system used is a good one, then the estimate obtained is likely to differ little from the true value. It is the purpose of the present manual to outline methods of developing good sampling systems for the quantities of major interest in fisheries biological research.

1.2 Elementary statistics

Before further discussion of sampling, it is necessary to describe some of the basic statistical concepts used in sampling theory. Statistics deal with the numerical properties of sets (or populations) of objects. Such populations can be real biological populations, e.g. the Rastrelliger in the Indian Ocean, or some other set of definite measurements, e.g. a set of temperatures, or even the possible estimates of a quantity (e.g. mean length of fish) obtained from a definite sampling system. That is, a quantity derived from one population (the lengths of fish) can be a member of another population (the estimates of the mean length). Each member of the population has a numerical value, the variate, which can take a range of possible values (e.g. length of fish). Such a population may be described by its frequency distribution; that is the frequency with which each of the possible values occurs. A distribution may be either discontinuous, when only certain values are possible - e.g. the throw of a dice can be only one of the values 1, 2, 3, 4, 5 or 6 - or continuous, when all values, at least within a range, can occur, e.g. lengths of fish. This distribution is often represented graphically, either as a histogram or frequency polygon, and for this a continuous variable will usually be grouped. Then for a frequency polygon the height of each point will give the frequency for a certain value or interval (for which it will normally be plotted at the mid-point of the interval); for a histogram the area of each section gives the total frequency in that interval, which will allow different class intervals to be used, e.g. 1 cm groups for small fish and 5 cm for larger fish.

Example 1.2.1

The following length frequency distribution of North Sea cod has been derived from Russell's data. Plot the data as a frequency polygon and as a histogram, first as given (1 cm intervals) and using groupings of e.g. 2 cm, 3 cm, 5 cm, 10 cm and 20 cm.

Length (l)	Freq. (n)	l	n	l	n	l	n	l	n	l	n
25	2	39	18	53	15	67	-	81	-	95	-
26	7	40	15	54	8	68	-	82	2	96	-
27	8	41	13	55	6	69	-	83	1	97	-
28	9	42	13	56	11	70	1	84	-	98	-
29	13	43	19	57	7	71	1	85	-	99	-
30	12	44	19	58	4	72	-	86	1	100	-
31	9	45	21	59	5	73	1	87	1	101	-
32	15	46	13	60	1	74	-	88	-	102	1
33	7	47	19	61	2	75	-	89	-	103	-
34	7	48	21	62	1	76	-	90	-	104	-
35	5	49	8	63	2	77	1	91	-	105	-
36	12	50	22	64	-	78	1	92	1		
37	13	51	18	65	-	79	-	93	-		
38	16	52	18	66	2	80	1	94	-	Total: 449	

Also plot as a histogram using coarser groupings (e.g. 10 cm, as against 2 cm) for fish over 60 cm.

Compare the effect of using different base points for grouping, e.g. using for the 5 cm groups 25-29, 30-34 etc., or 27-31, 32-36, etc.

The plots should be made on the same effective scale; when using say 2 cm intervals, the frequency in each interval will be about double that when using 1 cm, so that the scale used should be halved, i.e. for 1 cm intervals, 1 fish (n = 1) equals say 10 units on the graph paper, then for 2 cm intervals, 1 fish equals 5 units, and for 5 cm intervals 1 fish equals 2 units. The length scale should be unaltered. Plotted in this way the various polygons and histograms should be nearly identical.

This example illustrates also the problem of the correct choice of class interval. Here the 1 cm interval is too fine, and causes much extra labour in calculation, writing etc. The figures given are only a sample of much more extensive data, from which it may be easily seen that the irregularities extending over one or two 1 cm groups in the table given are purely chance effects. The possible exception is the peak at 50 cm, and low frequency at 49 cm; this exists, though is much less marked, in the complete data. This is probably due to the common recording error of rounding off measurements (often unconsciously); in this case several fish whose lengths actually were 49 (or 51) cm were recorded as 50 cm.

The best grouping to take - probably in the sample 3 cm or 5 cm - will depend on the data concerned; the more extensive the data and complex the frequency distribution the more numerous and finer will be the intervals. A fair guide is that the normal maximum number of intervals is around 20, and that except for a few intervals at the tails of the distribution, the numbers in each interval should not fall much below 10.

Example 1. 2. 2

The table below (from Fitch 1958) gives the length composition of Pacific Mackerel caught off California in the years 1956-57.

l = length, in quarter-centimetres, n = number of fish									
l	n	l	n	l	n	l	n	l	n
80		95	6	110	25	125	19	140	13
81		96		111	24	126	26	141	16
82		97	9	112	24	127	13	142	15
83	1	98	6	113	28	128	22	143	8
84		99	10	114	31	129	17	144	5
85		100	21	115	19	130	24	145	3
86		101	13	116	24	131	20	146	11
87	1	102	14	117	25	132	14	147	2
88	1	103	16	118	30	133	18	148	6
89	2	104	22	119	30	134	27	149	
90	2	105	33	120	17	135	16	150	
91	1	106	24	121	28	136	20	151	2
92	3	107	21	122	31	137	15	153	1
93	1	108	23	123	16	138	16	154	1
94	6	109	31	124	28	139	13	156	1

Total 1011

Plot this data as a histogram;
Repeat using groupings of 1/2, 1 and 2 cm

It will be found that many frequency distributions, when plotted, will appear as single peaks with more or less extensive tails above and below the peak. The differences between distributions will appear mainly in differences in the position of the peak - e. g. the peak in example 1. 2. 1 lies between 40 cm and 50 cm, and in the extent of the spread on either side of the peak. One or more of three quantities may be used to define the position of the distribution. That most generally used is the arithmetic mean - or more simply just the mean, usually denoted by m.

Thus if ten fish are measured, lengths 15, 19, 17, 22, 14, 13, 18, 17, 16 and 18 cm then

$$\text{mean} = m = \frac{1}{10}\ (15 + 19 + 17 + 22 + 14 + 13 + 18 + 17 + 16 + 18)$$

$$= \frac{1}{10}\ (169) = 16.9$$

This can be written in a more general form, by denoting the length of the fish by x and adding a suffix to denote the length of a particular fish; in the example above $x_1 = 15$, $x_2 = 19$ etc., and we can write

$$m = \frac{1}{10}\ (x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots x_{10})$$

Further, we can denote by x_i the length of any, unspecified fish - the i^{th} fish, where i may have any of the values 1, 2, 3 ..... 10 and m = 1/10 Sum of x_i when i = 1, 2....10. This can be written in mathematical shorthand, using the symbol $\sum$ for Sum of, and putting the upper and lower limits of i at the top and bottom of the symbol $\sum$ as,

$$m = \frac{1}{10} \sum_{i=1}^{i=10} x_i$$

Just as the square root symbol $\sqrt{}$ is an instruction to take the square root of the quantity following the symbol, so is the symbol $\sum$ an instruction about dealing with the quantities following after it. It tells you to add up a set of quantities, all of the same nature, of which the expression following the $\sum$ sign is a typical member. The numbers above and below the $\sum$ symbol tell you what particular quantities to add up.

The expression can be shortened further by omitting the i's from the limits when, as usually happens, it is clear that it is i that is taking the values from 1 to 10

$$m = \frac{1}{10} \sum_{1}^{10} x_i$$

or even shorter when the limits are also clear

$$m = \frac{1}{10} \sum x_i$$

Similarly, if we want to write an expression for the mean not of 10 items in particular, but of any, unspecified number, say n (when in the example n = 10) it is

$$m = \frac{1}{n} \sum_{i=1}^{i=n} x_i$$

or

$$m = \frac{1}{n} \sum_{1}^{n} x_i$$

or

$$m = \frac{1}{n} \sum x_i \quad \ldots\ldots\ldots\

of each peak (e.g. the lengths of each year-class). The mode (or modes) is most easily determined by plotting the data as a histogram or frequency polygon and reading the values from a smooth curve drawn through the plot. It is greatly affected by random variations in the data, so that a rather large sample is required to determine the mode with any precision.

The median is less affected by random errors than the mode, though more than the mean, the latter giving the most precise measure of the position of the distribution. However, particularly when the data are not grouped, or grouped rather finely, the median can be estimated extremely quickly. If the data are not grouped, the estimate of median is given by the central member if the number in the sample is odd, or midway between the two central members if the number is even. For grouped data a rough estimate of the median is given by the midpoint of the interval in which the central member lies; more precisely it is given by the proportion along the interval. For example in a sample of 101 fish, 40 are less than 16 cm long, 15 lie in the 16-17 group, and 46 are over 17 cm. The central member is then the 10th smallest in the 16/17 cm group, and the estimate of the median is $16 + \frac{10}{15} = 16.7$ cm.

Example 1.2.3 Estimate the mean, median and mode of the distribution in Ex. 1.2.1. Compare the time required to estimate each quantity.

$$\text{Mean} = \frac{1}{449}(2 \times 25 + 7 \times 26 + \ldots + 1 \times 102)$$

$$= \frac{1}{449} * 19877 = 44.28 \text{ cm}$$

Median is at the 225th fish counting from smallest up, or biggest down, and therefore lies in the 44 cm group. If the measurements of the fish have been recorded to the nearest cm so that this group contains fish between 43.5 and 44.5 cm, then a more precise estimate of the median is

$$43.5 + \frac{12}{19} = 44.1 \text{ cm}$$

(Note that two estimates of the median can be made, counting from both the smallest up, and the largest down and these should be checked against each other.

A definite mode lies near 30 cm with others probably around 40 and 50 cm.

Example 1.2.4

Repeat Ex. 1.2.3, using the data of 1.2.2 both as given and in $\frac{1}{2}$ cm and 1 cm groups. Compare your estimate of the position of the modes with the fact that the original data consisted of 6 year-classes of fish, the mean lengths of the year-classes being approximately 21 cm (only 4 fish), 27.5 cm, 31 cm, 33.5 cm and 38 cm (only 3 fish).

The mean (or the mode or median) tells us the position of the distribution - what is the average, (or most frequent or central) value of the individuals, i.e. the length of the cod in example 1.2.1 are centred around 50 cm. We also want to know how these lengths vary about this central value -are all the lengths of the fish between 49 and 51 cm, or do they, as in the example, vary between 25 and 100 cm, or even say between 5 and 150 cm. If we take the differences between the individual values and the mean some will be positive,

and some negative, and the average value will be about zero. We therefore take the square of the difference between the individual value and the mean, and the average value of this squared difference is called the variance. For example a group of ten fish taken from the population of Ex. 1.2.1 have lengths

35, 38, 40, 44, 45, 47, 50, 52, 53 and 66 cm

Their mean length is 47.0 cm; the individual differences from the mean are

-12, -9, -7, -3, -2, 0, +3, +5, +6 and +19 cm

so that the variance of the population formed by the lengths of this small group of fish is

$$\frac{1}{10}(144 + 81 + 49 + 9 + 4 + 9 + 25 + 36 + 361) = \frac{1}{10} \times 714$$

$$= 71.4$$

The square root of the variance, which is in some ways an average value of the deviation from the mean, is called the standard deviation. The standard deviation of the population above is 8.45 cm.

In mathematical terms the variance is generally denoted by S^2 and the formula for the variance is

$$S^2 = \frac{1}{N}\sum^{N} (x_i - M)^2 \quad \ldots\ldots\ldots\

$$= \sum_{i=1}^{n} (x_i^2 - 2Mx_i + M^2 + 2(M-m)x_i + m^2 - M^2)$$

$$= \sum_{i=1}^{n} (x_i - M)^2 + 2(M-m)x_i + m^2 - M^2)$$

This expression is merely the addition of a number of terms, and by grouping them together in a slightly different order this expression becomes

$$\sum_{i=1}^{n} (x_i - M)^2 + \sum_{i=1}^{n} 2(M-m)x_i + \sum_{i=1}^{n} (m^2 - M^2)$$

In the second term the factor 2 (M-m) is common to all the items in the addition, and so can be taken outside the sign; in the third term all the items are the same, and there are n of them, so the expression may be written

$$\sum_{i=1}^{n} (x_i - M)^2 + 2(M-m) \sum_{i=1}^{n} x_i + n(m^2 - M^2)$$

$$= \sum_{i=1}^{n} (x_i - M)^2 + 2(M-m)nm + n(m^2 - M^2)$$

$$= \sum_{i=1}^{n} (x_i - M)^2 - n(m^2 - 2Mm + M^2)$$

$$= \sum_{i=1}^{n} (x_i - M)^2 - n(m - M)^2$$

$(m-M)^2$ being squared must be positive (or possibly zero if m is exactly equal to M), so that $\sum (x_i - m)^2$ will always be less than, or equal to $\sum (x_i - M)^2$. Therefore $\frac{1}{n}(x_i - m)^2$ will be less than $\frac{1}{n}(x_i - M)^2$ and so would give a biassed and low estimate of the variance. It can be increased slightly by dividing not by n, but by n - 1, and this slight decrease in the divisor can be shown mathematically to balance exactly the degree of bias; that is, for an unbiassed estimate of the variance we have

$$s^2 = \frac{1}{n-1} \sum_{1}^{n} (x_i - m)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.3)$$

The formulae for mean and variance can be written in several ways for greater simplicity of computation: thus rewriting the formula for the variance we have

$$(n-1)s^2 = \sum_1^n (x_i - m)^2 = \sum (x_i^2 - 2mx_i + m^2)$$

$$= \sum x_i^2 - 2m\sum x_i + nm^2$$

$$= \sum x_i^2 - nm^2$$

which can also be written as $(n-1)s^2 = \sum x_i^2 - m\sum x_i$

or $$(n-1)s^2 = \sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.4)$$

The form of 1.4 is suitable for computation particularly on a calculating machine which permits rapid calculation of sums of squares. Suitably arranged, the computations can be to a fair extent self-checking. First calculate $\sum x_i$ and the mean. Then enter x_1 on the register and square it, so that on the results register appears x_1^2, and on the multiplier register, x_1. Do not clear these entries, but enter x_2 and square it so that on the results and multiplier appears $x_1^2 + x_2^2$ and $x_1 + x_2$ respectively. Repeat for the n numbers, finally obtaining $\sum x_i^2$ and $\sum x_i$; check that $\sum x_i$ agrees with the value already obtained. This will detect most of the likely errors - e.g. omitting or misreading one value of x.

The calculations may be considerably simplified and the chances of error reduced, by taking some convenient transformation of the raw data. For instance, suppose the monthly landings of fish during 6 months were 75, 67, 82, 73, 69 and 71 tons; instead of calculating 75^2 etc., we may taken an arbitrary origin at 70 say, and compute the mean and variance of 5, -3, 12, 3, -1 and 1 (2.83 and 28.17 respectively). Converting back to the original scale, the mean of the original distribution is therefore 2.83 + 70 = 72.83 tons. The variance is unchanged by change of origin so that the variance in the original scale is also 28.17, and the standard deviations s = 5.31.

The above catches might have been expressed in a different scale, as 75,000..... 71,000 kilogrammes. Expressed as tons, with origin at 70 tons, we get as before a mean of 2.83, variance 28.17 and standard deviation 5.31. Converting back to kilogrammes we have to add 70 (to get correct origin) and multiply by 1,000 (to get correct scale), and the mean monthly landings are 72,833 kilogrammes.

This is a simple example of a transformation which shows that instead of making calculations (sums, sums of squares) concerning the values x, which may be large, we can make the calculations using another set of values y, obtained from x by some direct relation, y = f (x). The simplest is a change of origin, so that

$$y = x - a \text{ (as in the first example, where a = 70)}$$

The means and variances are given by the simple relation, which can be deduced at

once from equations 1.1 and 1.2

$$\text{mean } x = M_x = M_y + a$$

$$\text{variance of } x = S_x^2 = S_y^2$$

Another simple transformation is a change of scale.

$y = bx$ (as in the second example, where b equals one thousandth)

$$\text{Thus } M_x = \frac{1}{b} M_y$$

$$S_x = \frac{1}{b} S_y$$

$$S_x^2 = \frac{1}{b^2} S_y^2$$

The two transformations may be combined, $y = b(x-a)$

$$M_x = \frac{1}{b} M_y + a$$

$$S_x^2 = \frac{1}{b^2} S_y^2$$

Example 1.2.5 The weights of fish landed at Rameswaram Island, south India, during the 12 months July 1953 - June 1954 were 205, 218, 150, 136, 89, 55, 112, 28, 93, 105, 186, 253 tons (data from Krishnamurthi 1957).

Calculate the mean monthly landing, and the variance and standard deviation of the monthly landings; check that the range (253-28 = 225 tons) is about 3.2 times the standard deviation.

If the data are arranged in class intervals, e.g. length compositions giving the number in each centimetre group, the arrangement of the calculations for mean and variance are slightly different. The value of each class, e.g. its mid-point, must be included f times, where f is the number of individuals in the class. Thus equations 1.1 and 1.4 must be re-written as

$$m = \frac{1}{n} \sum_{i=1}^{k} f_i x_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.5)$$

$$s^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{k} f_i x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{k} f_i x_i\right)^2\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.6)$$

where k = number of classes

$$n = \text{number of individuals} = \sum f_i$$

A change of origin before starting computation is likely to be particularly valuable in these calculations; the methods are best described by an example showing the calculation of the mean and the variance of the lengths of Rastrelliger sampled on Bangkok market on 31st October 1958. The working origin is taken at 17.5 cm, and the working units as half-centimeters.

Length group (cm)	Frequency	New Scale		
x_i	f_i	y_i	$f_i y_i$	$f_i y_i^2$
15.5	8	-4	-32	128
16.0	7	-3	-21	63
16.5	4	-2	-8	16
17.0	2	-1	-2	2
17.5	8	0	$\sum_- f_i y_i = -63$	
18.0	11	+1	11	11
18.5	2	+2	4	8
19.0	3	+3	9	27
19.5	1	+4	4	16
	n = 46		$\sum_+ f_i y_i = 28$	$\sum f_i y_i^2 = 271$

The origin has been chosen near to the probable mean of the distribution - a good choice of origin will reduce the computational work, but a difference of one or two groups either way will not affect work much. The column of values of $f_i y_i$ are obtained by multiplying the second and third columns together, and the values of $f_i y_i^2$ can be obtained by multiplying again by y_i, without computing y_i^2 as such. In calculating the mean the positive and negative values of $f_i y_i$ are added separately, then

$$m_y = \frac{1}{n}\left(\sum_+ f_i y_i - \sum_- f_i y_i\right) = \frac{1}{46}(28-63) = -\frac{35}{46} = -0.76$$

$$\text{var} = \frac{1}{n-1}\left(\sum f_i y_i^2 - \frac{1}{n}\left(\sum f_i y_i\right)^2\right) = \frac{1}{45}\left[271 - \frac{35 \times 35}{46}\right] = \frac{244.4}{45} = 5.4$$

$\therefore$ In the original scale, mean length = 17.5 - 0.38 = 17.12 cm, variance = $\frac{5.4}{4}$ = 1.4 cm^2, standard deviation = 1.2 cm.

If the class intervals are not units, so that the scales of x and y may be different, as in the sample above, care should be taken to convert back the results for mean and variance etc. to the proper scale.

All the results obtained by these computations should be checked with a thoroughness corresponding to the accuracy and precision needed in their application. Repetition of the same computations, besides being tedious, is also inefficient in detecting and eliminating errors, as they are not unlikely to be repeated. A good check where accuracy is required is to compute, not only $\sum x$ and $\sum x^2$, but also $\sum (x+1)$ and $\sum (x+1)^2$.

Thus, applied to the data of Ex. 1.2.1, we would compute

$$\sum x = 2x25 + 7 \times 26 + \ldots + 1 \times 102 = A$$

and $$\sum x^2 = 2x(25)^2 + 7x(26)^2 + \ldots + 1 \times (102)^2 = B$$

and also $$\sum (x + 1) = 2 \times 26 + 7 \times 27 + \ldots + 1 \times 103 = C$$

$$\sum (x + 1)^2 = 2x(26)^2 + 7x(27)^2 + \ldots + 1 \times (103)^2 = D$$

Each fish will account for a difference in 1 between A and C, and since there are 449 fish in all the first check, to ensure that A (and incidentally C) has been calculated correctly, is to see whether A + 449 = C. Similarly D should be bigger than B by a known amount equal to twice the sum of the lengths plus the number of observations

i.e. $D = B + 2 \times A + 449$

Example 1.2.6 Calculate for the data in Ex. 1.2.1 the quantities A, B, C, D above, and check that

(i) $C = A + 449$

(ii) $D = B + 2A + 449$

and similarly, for the data in Ex. 1.2.2 which refer to a total of 1011 fish calculate $\sum x$, $\sum (x + 1)$ and $\sum x^2$ and $\sum (x + 1)^2$ and check that

(i) $\sum (x + 1) = \sum x + 1011$

and (ii) $\sum (x + 1)^2 = \sum x^2 + 2 \quad x + 1011$

These are special cases of the general rules, which will always be followed if no mistakes have been made in the calculations, that

(i) $\sum_{1}^{n} (x_i + 1) = \sum x_i + n$

$$(ii)\sum_{1}^{n}(x_i + 1)^2 = \sum x_i^2 + 2\sum x_i + n$$

where n is the number of observations.

A simple check of accuracy (but not of precision) which should always be made is to compute the range of the distribution (greatest value minus least value), and to divide this by the standard deviation. The range will normally be between 3 and 6 times the standard deviation, being greatest when there is a single extreme value, and when the number in the distribution is large. It is worth distinguishing accuracy and precision. If a mean length of a number of fish is 43.26 cm, then an estimate of 43.18 cm is very precise and accurate, an estimate of 43 cm is not very precise, but accurate, 37.2 cm is precise, but inaccurate, and 35 cm both imprecise and inaccurate. Precision defines narrowness or otherwise of the limits within which the quantity by implication lies; thus an estimate of 37.2 would presumably include all values from 37.15 to 37.25. The precision is therefore related to the number of significant figures in quantity given. The accuracy is the closeness, or otherwise, of the estimate to the real value.

2 THEORY OF SAMPLING

2.1 Introduction

Every sampling system is used to obtain estimates of certain properties of the population being studied, and the sampling system will be judged by how 'good' the estimates obtained are. Individual estimates may, by chance, fall very close to, or differ greatly from, the true value, and give a poor measure of the merits of the system. A good sampling system will, on occasions, give an estimate which is far from the true value, just as a poor system may, very occasionally, give an estimate very close to the true value. A system is better judged by the frequency distribution of the many estimates which are, or could be, obtained by repeated sampling. A good system would give a frequency distribution with small variance, and mean estimate about the same as the true value. The difference between the mean estimate and the true value is called the bias. (The term 'bias' is also used for the process whereby the difference arises). The extent of bias and variance of sampling system are to a large extend independent - a system can give estimates having small variance, i.e. differ little amongst themselves, but with a large bias, so that all the estimates differ greatly from the true value. (A measuring board with the figures on the scale almost illegible would introduce some extra variance; a board with the scale displaced to one side would introduce a bias).

Example 2.1.1 Catches of mixed species of fish are examined by two observers for the percentage of one species of Leiognathus. Observer A works rapidly but carelessly, missing several fish of the required species; observer B works much more slowly but more carefully. From a succession of samples their estimates of the percentage of Leiognathus splendens in the catch were

A.					B.				
4	4	3	5	4	9	7	11	4	8
3	5	4	6	4	4	10	8	9	12
6	3	4	3	5	8	3	6	10	15
4	5	4	4	6	11	12	7	13	11
5	3	5	4	5	10	5	8	9	12

By calculating the means and variances of the above distributions show (i) that the estimates obtained by A are more precise (have less variance) than those of B (0.89:9.03); (ii) that if, from other data, it is known that the true percentage was 9.1, then A has a strong negative bias; (iii) if it is known that A misses half the fish, a relatively unbiassed and precise estimate can be obtained by doubling the estimates obtained by A (mean 864, bias (i.e. the difference between the estimated mean and the true mean) - 0.46, variance 3.6)).

Bias may arise from a poor method of analysis, but more often from a poor choice of samples, or from the method whereby the measurement or counts are made or the samples are obtained , e.g. if the fish are sorted when landed, and the samples are mainly taken from the larger categories, the average size will be over-estimated - a positive bias in mean size - or if plankton hauls are made with a coarse meshed net the numbers of small diatoms will be under-estimated - negative bias in mean numbers of small diatoms, but a positive bias in mean diatom size. Bias in some form often occurs, particularly when attempting to obtain samples typical of the true condition in the sea, whether of diatoms with a plankton net, or of fish with a trawl.

If the size of sample increased, or the data of two or more samples combined, then the bias will remain unaltered, but the variance will be reduced, roughly in inverse proportion to the size of sample, or number of samples taken. The latter will in turn be closely related to the amount of work, or cost, involved in the sampling programme. Any degree of precision, *i.e. a variance as small as desired, can in theory at least, be obtained by taking large enough samples. The aim of good sampling is therefore not so much to obtain a given level of precision (small variance) but to do so with the least cost. Bias however, cannot be reduced purely by increased sampling, nor can its presence often be detected by subsequent analysis of the data (c.f. Ex. 2.1.1 where there is nothing in the data themselves to tell which of samples A or B are biassed). Bias can normally only be detected and hence eliminated by careful examination of the whole sampling process from beginning to end. In most situations great care should be taken to ensure that all likely sources of bias have been eliminated. There are however some situations where the bias is easily measured and it is simpler to allow the bias to occur, and to remove it in subsequent analysis (for example gill nets are highly selective in the size of fish they catch, and hence provide a biassed sample of say, the mean length. However this selectivity can be measured and corrected in the later analysis). However in this situation, as in all others, the chances of bias occurring must be fully examined before sampling, and if bias is accepted, its extent must be carefully measured, independently of the sampling process.

Example 2.1.2 In the situation of example 2.1.1, larger samples are obtained by combining 5 of the original samples. By taking the mean of the rows as the estimate for the larger sample,shows:

(i) that the variances of both sets have been reduced (that of A from 0.89 to 0.52, that of B from 9.03 to 1.29)

(ii) that the extent of the bias of A's samples is unaltered (mean is unaltered)

The considerations above (bias eliminated, or at least known and measured, and variance a minimum for a given amount of sampling) will determine the sampling method, but the amount of sampling will be determined by the precision required. Usually it is not possible to state exactly the degree of precision required, but two limits can usually be given. At the lower limit the variance is so great that the information given by the sample is no practical value - the size of sample must be increased, or the sampling procedure abandoned. The estimates obtained from a single sampling scheme are usually combined with other data, some perhaps from other sampling systems, most of which will have a greater or lesser variance. The variance of the final answer will depend on variance of all the constituent pieces of data, but chiefly on the variance of the least accurate parts - a chain is as strong as its weakest link. For example the total catch of a fleet may be estimated from data of the average catch per landing times the total number of landings. If the number of landings is only known within $\pm$ 10%, then however good the information

*

A useful distinction will be maintained here between precision and accuracy; this corresponds closely to the distinction between variance and bias (or rather their reciprocals). A precise figure will have little variance and will be given with many significant digits, but may depart rather widely from the time value. If a fish is truly 17.638 cm. long, precise measures of its length would be 17.64 cm, or 18.32 cm, but the latter is grossly inaccurate. Accurate but less precise measures are 17.6 cm. or 18 cm.

on the average catch per landing, the total quantity landed will only be known within ± 10% at best. Once a certain level of precision has been achieved in a single sampling scheme, further improvement will not improve the precision of the final result, and the effort (time, manpower, etc.) would be better employed in improving the precision of other data.

2.2. Random sampling

The basic concept in all sampling is the random sample. A sample of objects from a population is random if all the members of the population have an equal chance of appearing in the sample. It is very important to remember that this applies to all members of the population, exceptional as well as typical members. For instance, the whiting landed by any one ship at Lowestoft will usually (and here it will be assumed always) have a smooth unimodal length composition, with the mode usually between 28 and 30 cm, but occasionally, say once in 30 times, as high as 35 cm. A single sample of whiting from one ship if taken at random must therefore occasionally (once in 30 times on an average) give a mode at or above 35 cm, but usually around 28 to 30 cm. If then a fishery biologist, relying on one sample, obtained a mode at 35 cm, this departure from the average of 29 cm does not necessarily indicate non-randomness because such misfortune will occur one in 30 times. The answer is to take more samples - all of three samples will have modes over 35 cm only once in 27, 000 times.

2.2.1 Random numbers

A useful and widely applicable method of obtaining a truly random sample is by use of random numbers, as described in most statistical books. The individuals in the population from which a sample is to be drawn are allotted numbers, and those to be sampled are determined by reference to a table of random numbers. For instance, if a sample of 5 has to be taken from a population of 100, and the first five random numbers are 3, 47, 43, 73, and 86, the individuals corresponding to those numbers will be sampled. If the number of individuals in the population is not exactly 100 (or 1, 000 etc) some of the random numbers occurring will not correspond to numbers of the population and will have to be discarded. This wastage can be reduced if two or more numbers are ascribed to each individual, provided each individual has the same i.e. has an equal chance of occurring in the samples. Suppose, for example, 5 units have to be sampled from a population of 24; in this case to each individual 4 numbers would be allocated - the first unit having for instance numbers 01 to 04 etc., the 24th having 93-96, so that only the numbers 97-100 are not used. The individuals to be sampled, corresponding to the previous set of five random numbers, would then be numbers 1, 12, 11, 16 and 22 (if one of the random numbers is 97 or over, it is discarded and another one taken). Instead of choosing all the units in the sample individually from the table of random numbers, units may be taken at regular intervals e.g. every fifth or hundredth unit, and only the first chosen by use of random numbers. In the first example, one-twentieth of the population is to be sampled, so that the sampling interval would be 20. The first random number is 3, so the complete sample would be the units numbered 3, 23, 43, 63, 83. Such a system can be dangerous when there is any natural periodicity in the population corresponding to the interval of sampling. For example sampling at a landing place to obtain total catch should not be done every seven or fourteen days when there are great systematic differences between landings on different days of the week.

Example 2.2.1 Fish are landed at a certain landing place throughout the year. The total landings are to be estimated by sampling the catches in 30 days during the year. Determine the days to be sampled by using random numbers:

(i) directly from sets of random numbers 000 to 999, by numbering the days 1 to 365.

(ii) by giving each day 2 numbers, 1 and 2 to 729 and 730.

(iii) by giving each day 27 numbers 1-27 up to 9829-9855, using random numbers 0000 to 9999.

(iv) by sampling every 12 days the starting day being chosen at random from days 1-12; (some samples may have 31 days).

If random numbers, or a similar randomizing process are not used, then it is likely that all individuals in the population will not have equal chances of appearing in the sample. If there is any correlation between the quantity being measured and probability of appearing in the sample, the result may be biased, perhaps strongly. For instance at a busy fish market it is often convenient when sampling the landings of a particular boat to work on the fish landed first. These will tend to be the fish caught last, and therefore the freshest. Such a sample would be heavily biased if used to estimate the freshness of the fish landed. The fish caught last will not however be very likely to differ in size from the other fish, so sampling the fish landed first may give unbiased estimates of the average size. The absence of bias must not be assumed too readily, and the possibility of bias arising must be carefully investigated. In the example above bias could arise if ships tended to do some fishing near their home port at the end of the trip, and if the size of fish on these grounds is different from the average. These and other sources of possible bias can only be detected and eliminated if there is a full knowledge and understanding of the fishery - how the fish are caught, how they are handled on board, and the arrangements of the fish market.

The precision of estimates obtained by true random sampling is readily determined. If a population is being sampled for some characteristic (e.g. vertebral count), which in the population has mean M, variance S^2, and a sample of n independent random individuals are taken having values $x_i \ldots\ldots x_n$, the resulting estimate of the mean value in the population is

$$\bar{x} = \frac{1}{n}\sum x_i, \quad \ldots\ldots\ldots\

2.3 Stratified sampling

When sampling a heterogeneous population the precision achieved can be increased - sometimes very greatly - and the risk of bias reduced by dividing the population into sections, each relatively homogeneous, and sampling each section (or stratum) separately. Each stratum is then sampled independently, and the estimates for each obtained. These can then be combined to give the estimate for the whole population. The variance of this estimate will also be obtained by combining the variances of the estimates within the individual strata. As these latter will tend to be small - the strata being relatively homogeneous, so that the variance within strata is less, and possibly much less than the variance in the population as a whole - the variance of the final, combined estimate will also be small.

In mathematical terms suppose the population consists of N individuals, N_i in the i^{th} stratum where $N = \sum N_i$ and a sample of n_i is taken from the i^{th} stratum have values of the quantity to be estimated (length of fish, weight caught etc.), equal to y_{ij}, $j = 1 \ldots n_i$ the estimated mean value $\bar{y}_i$ in the stratum is

$$\bar{y}_i = \frac{1}{n_i} \sum_j y_{ij} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

and an unbiased estimate of the mean value in the whole population is given as the weighted mean of the means of the individual strata, the weighting factor being the total numbers in the strata,

$$\text{i.e.} \quad \bar{\bar{y}} = \frac{1}{N} \sum_i N_i \cdot y_i$$

If the variance within the i^{th} strata is S_i^2

$$\text{var}\,(\bar{y}_i) = \frac{1}{n_i} S_i^2$$

$$\text{and var}\,(\bar{\bar{y}}) = \frac{1}{N^2} \sum \frac{N_i^2}{n_i} S_i^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

provided n_i is small compared with N_i. Otherwise the variances become

$$\text{var}\,(\bar{y}_i) = \frac{1}{n_i} \left(1 - \frac{n_i}{N_i}\right) S_i^2$$

$$\text{var}\,(\bar{\bar{y}}) = \frac{1}{N^2} \sum N_i^2 \frac{1}{n_i} \left(1 - \frac{n_i}{N_i}\right) S_i^2$$

$$= \frac{1}{N^2} \sum N_i (N_i - n_i) \frac{1}{n_i} S_i^2$$

This may be compared with the variance of the estimate obtained by random sampling from the whole population, which is

$$\text{var}\,(\bar{y}') = \frac{1}{n} S^2$$

or $$\text{var}\,(\bar{y}') = \frac{1}{n}\left(1 - \frac{n}{N}\right) S^2 \quad \text{if n is not small compared with N where}$$

S^2 is the variance in the population as a whole.

Example 2.3.1

The catch of a commercial trawler landing haddock at Aberdeen was sorted into four size categories forming the four strata. Samples of haddock from each category were measured, and the resulting data can be summarized as follows:

Category	N_i	n_i	$\sum y_{ij}$	$\sum y_{ij}^2$
Small	2432	152	5284	185532
Small-medium	1656	92	3817	158953
Medium	2268	63	3033	146357
Large	665	35	2027	118169
Total	7021	342	14161	609011

Where y = length of fish in cm

From these figures, using as our estimate of S_i^2 the usual quantity

$$s_i^2 = \frac{1}{n_i - 1}\left[\sum y_{ij}^2 - \frac{1}{n_i}\left(\sum y_{ij}\right)^2\right]$$

we have:

Category	$\bar{y}_i$	$N_i \bar{y}_i$	S_i^2	S_i^2/n_i	$N_i^2 S_i^2/n_i$
Small	34.763	84,544	12.21	0.0803	474900
Small-medium	41 489	68,706	6.47	0.0703	192800
Medium	48.143	109,188	5.48	0.0870	447500
Large	57.914	38,513	22.85	0.6529	288700
		300,951			1,403,900

and hence $\bar{\bar{y}} = \frac{300951}{7021} = 42.9$

and var $(\bar{\bar{y}}) = \frac{1403900}{(7021)^2} = 0.0285$

and s.d.$(\bar{\bar{y}}) = 0.17$

The 95% confidence limits for the true mean length of the catch are therefore 42.9 ± 0.3, i.e. 42.6 - 43.2 cm.

The data can also be used to give a rough measure of the variance of the estimate obtained from a random sample of 342 from the whole catch. For this we shall take as the estimate of S^2, the variance of the population as a whole

$$341.s^2 = \sum y_{ij}^2 - \frac{1}{n}\left(\sum y_{ij}\right)^2 = 609011 - \frac{1}{342} \times 14161^2$$

therefore s^2 = 66.4 (compared with the largest within strata variance of 22.85)

$$\text{var } (\bar{y}') = \frac{66.4}{432} = 0.197$$

$$\text{s.d. } (\bar{y}') = 0.44$$

The estimate of S- used is of course not entirely correct, as the sample used is far from being a true random sample, the medium fish being under-represented. However it is sufficiently accurate to indicate the great reduction in variance in using stratified sampling, - in this example a reduction to about one-seventh of the unstratified variance, equivalent to an increase in sample size of seven times.

The benefit of stratified sampling will be increased by sampling the various strata in the best proportion. Those strata containing many individuals, or which are highly variable need more sampling than small or uniform strata. The variance will be a minimum for a given total sample size, n, if

$$n_i \propto N_i \times S_i$$

or $\frac{n_i}{N_i} \propto S_i$, i.e. the proportion sampled within a strata is proportional to the variance in that strata. If n_i is not small compared with N_i this formula does not hold exactly, but will be close enough to give a good guide to the best allocation.

Example 2.3.2.

In the example 2.3.1., determine the best allocation to each strata of the total number of fish sampled (342), and using the values of S^2 calculate the variance of the estimated mean length for that allocation of samples.

Example 2.3.3.

Along a certain coast the 100 places where fish are landed can be roughly graded into three classes according to the weight of fish landed. During one week the weights landed were:

Large landing places:	45,	59,	87,	41,	71,	25,	9,	69,	10,	7.
Medium:	17,	13,	19,	26,	1,	8,	27,	11,	12,	26
	5,	8,	10,	16,	16,	4,	16,	16,	13,	29
	14,	25,	29,	27,	20,	25,	2,	7,	3,	12
Small:	2,	6,	7,	0,	1,	2,	1,	5,	4,	7
	8,	9,	3,	2,	5,	4,	2,	0,	2,	8
	5,	3,	8,	9,	8,	9,	1,	6,	5,	3
	3,	4,	7,	5,	5,	3,	2,	4,	6,	1
	6,	2,	5,	1,	0,	3,	8,	0,	4,	3
	3,	5,	5,	0,	7,	0,	9,	7,	9,	0

By calculating the variance within each class and in the population as a whole determine what would be the best method of estimating the total weekly catch along the whole coast, if the catch at only 20 places (one in five) can be determined (e.g. by visiting the landing places). What is the variance of this estimate, and how does it compare (a) with that of a simple random sample from the whole population, and (b) using stratified sampling, taking a sample of one-fifth from each class.

2.4. Sub-sampling, or two-stage sampling.

When the population being sampled is at all extensive or complex, the practical problems in taking a simple random sample are great, and the time taken for even a small sample may be large. The time required for a given number in the sample may be greatly reduced by carrying out the sampling in two stages. First the complete population may be divided into a number of distinct primary units or sub-populations, and from these a sample is taken. From each of these sampled sub-populations a secondary sample, or sub-sample of individuals is taken. For instance, in estimating the total catch along a certain coastline, the basic unit may be taken as the landing of an individual boat. Taking landings at random along the whole coastline would mean an impossibly large amount of travelling; landings at a particular place on a particular day from a convenient primary unit. The procedure would then be to select (e.g. by random numbers) certain landing places on certain days, and at these selected places sample certain of the boats landing there.

Sub-sampling can of course be taken to more than two stages. If in the above example detailed examination, e.g. of size or maturity, was required, this might be done on a sample of a box of fish (or even a sub-sample from a box) landed by a certain boat at the landing place, giving three (or four) stages of sampling.

The disadvantage of sub-sampling is of course that individuals in the same primary unit are likely to be more nearly the same than individuals in the population as a whole. Thus, after examining one individual in the unit, e.g. weighing the catch of one boat landing at a particular place, examination of further individuals from that unit will tell one less about the characteristics of the whole population (e.g. the average catch per boat at all landing places), than examination of individuals from different primary units. This has to be balanced against the increased number of samples that can be taken in a given time by using two-stage sampling. In general terms, if individuals within a primary unit are very variable it is best to take many samples within a unit, with comparatively few primary units. Conversely if the variation within a unit is small, but there are considerable differences between units, then a large number of primary units should be sampled, with a small number of individuals sampled in each.

The method can be illustrated in mathematical terms, supposing for the sake of simplicity that the population can be split into K primary units, each of N individuals, and k primary units are sampled, a sub-sample of n individual being taken from each.

Then if M is the population mean, and M_i the mean for the i^{th} primary unit, we have as the estimate of the mean of any sampled primary unit.

$m_i = \frac{1}{n} \sum_{j=1}^{n} x_{ij}$ where x_{ij} is the value of the j^{th} individual in the i^{th} unit.

and the estimate of the population mean is

$$m = \frac{1}{k} \sum_{i=1}^{k} m_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

Then the variance of m_i about M_i is $1/n \times S_w^2$, where S_w^2 is the variance of the individuals of the i^{th} primary unit about the unit mean. The variance of the estimated population mean will be made up of two parts - the variance of the estimated unit means about the true unit means, and the variance of the latter about the population mean; that is:

$$\text{var}\ (m) = \frac{1}{k} S_B^2 + \frac{1}{k} \cdot \frac{1}{n} \cdot S_w^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.5)$$

where S_B^2 is the variance of the unit means about the population mean. An unbiassed estimate of the variance of m is:

$$s_m^2 = \text{estimate of } S_m^2 = \frac{1}{k} \frac{1}{(k-1)} \sum_i (m_i - m)^2$$

$$s_m^2 = \frac{1}{k(k-1)} \left[\sum_i m_i^2 - \frac{1}{k} \left(\sum_i m_i \right)^2 \right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

Example 2.4.1 (from Pope, 1956)

A random sample of herring was drawn from the total number of landings in a week, and 50 herring from each selected landing were taken at random and measured. The following data were obtained:

Ship	1	2	3	4	5
Sum $\sum_j x_{ij}$	1244.3	1324.2	1335.4	1299.7	1270.5
Sum of Squares: $\sum_j x_{ij}^2$	31020.97	35127.08	35730.30	33900.99	32558.55

Estimate the mean length of herring in the week's landings, and its standard error. First we obtain the mean for each ship, equal to 24.9, 26.5, 26.7, 26.0 and 25.4. Therefore the estimates required are given by

$$m = \frac{1}{5} (24.9 + \ldots + 25.4) = 25.9$$

$$s_m^2 = \frac{1}{5 \times 4} \left[(24.9)^2 \ldots + (25.4)^2 - \frac{(24.9 + \ldots + 25.4)^2}{5} \right] = 0.113$$

$$s_m = 0.34$$

The variances within and between primary units can also be estimated separately. Within any primary unit we have an estimate of S_w^2 as

$$(S_w^2)i = \frac{1}{n-1} \sum_{j=1}^{n} (m_i - x_{ij})^2$$

$$= \frac{1}{n-1} \left(\sum x_{ij}^2 - \frac{1}{n} \left(\sum x_{ij} \right)^2 \right)$$

These estimates from separate primary units can be combined to give as the best estimate

$$S_w^2 = \frac{1}{k} \frac{1}{n-1} \sum_{i=1}^{k} \sum_{j=1}^{n} (m_i - x_{ij})^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.7)$$

From equations (2.5) and (2.6), the between units variance can be deduced from the equation

$$(k-1)\left(S_B^2 + \frac{1}{n} S_w^2 \right) = \sum_{i=1}^{k} (m_i - m)^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.8)$$

and from the value of S_w^2 given by equation (2.7)

Example 2.4.2. Calculate the within ship and between ship variance of herring lengths from the data of Example 2.4.1. We have as the estimate of within ship variance

$$5 \times 49 \times S_w^2 = (31020.97 - \frac{1}{50} \; 1244.3^2) + \ldots + \ldots$$

$$\therefore \quad 245 \, S_w^2 = 378.62$$

$$S_w^2 = 1.545$$

$$\text{Also} \quad S_B^2 + \frac{1}{50} S_w^2 = (24.9^2 + \ldots\ldots)^2$$

$$S_B^2 = 0.56 - 0.03 = 0.53$$

In the calculations of Examples 2.4.1. and 2.4.2. it may be seen that the major contribution to S_m^2, the variance of the estimated mean length of all fish landed, comes from S_B^2, the between ship variance. Further, it follows from equation (2.5) that the effect of this on the variance of the mean can be reduced by increasing k, the number of primary units sampled, but not by increasing n, the number of individuals sampled in each primary unit. The time spent sampling the herring landings would therefore most probably be more efficiently used if the number of ships sampled was increased, at the expense of reducing the number of fish measured, e.g. 6 samples of 30 = 180 fish, instead of 5 of 50 = 250 fish. The best allocation of time will be determined by the time taken in the various stages of sampling, as well as on the variances concerned. The total time spent can to a first approximation, be split into three parts.

(a) The overhead time; the time spent in preparation, including time taken travelling to the sampling area from headquarters. This time is more or less fixed irrespective of the amount of sampling.

(b) The time between primary units, - in the example the time spent moving from one ship to the next, - which will be proportional to the number of primary units.

(c) The time within primary units, - the time spent examining the individuals within each primary unit.

The total time spent will therefore be given by

$$t = t_o + k\,t_b + nk\,t_w \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

when

t_o = overhead time
t_b = time moving from one primary unit to another
t_w = time spent examining one individual.

The best allocation (i.e. that giving the minimum variance) of sampling time, in terms of the number of individuals sampled in each primary unit is given by

$$n = \frac{S_w}{S_b}\sqrt{\frac{t_b}{t_w}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

Example 2.4.3. Using the data of the previous examples, and assuming that 20 fish can be measured in a minute, and that the time taken to move from one ship to the next is 5 minutes, show that the least variance in the estimated mean lengths for a given amount of sampling is about 17 fish.

So far it has been assumed that the primary units are all of the same size. When, as usually happens, they are of different sizes it is important that the correct weights are applied to each unit. Then equation 2.4 must be re-written as

$$m = \frac{1}{N}\sum_{i=1}^{k} N_i m_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

where N_i = number of individuals in the i^{th} primary unit

$N = \sum N_i$ = total number in all sampled primary units

or as

$$m = \frac{1}{N}\sum_{i=1}^{k}\left[N_i \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.12)$$

where n_i is the number of individuals sampled in the i^{th} primary unit, which will not necessarily be the same for all primary units. If n_i is taken so that the sampling ratio n_i/N_i is the same for all units equal to p say then (2.12) reduces to

$$m = \frac{1}{N}\sum_i \frac{1}{p}\sum_j x_{ij}$$

$$\therefore\ m = \frac{1}{n}\sum_i\sum_j x_{ij} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.13)$$

where n is the total number of individuals sampled; this, of course, is a most convenient form for computation. The formula for the variance (equation 2.5) has also to be re-written, and becomes

$$\text{var } m = \frac{1}{N^2} \sum_{i=1}^{k} N_i^2 \text{ var } m_i$$

$$\text{where var } m_i = \frac{1}{n_i} S_w^2 + S_b^2$$

The formula (equation 2.10) for the best number to sample in each unit will also no longer be strictly applicable. The equation could be modified to give a formula determining precisely the best allocation to each primary unit sampled. However this formula will be rather cumbersome, and will need extra information on the variances within each primary unit (which may not be the same for each unit). The extra precision achieved may not be worth the effort involved, and a more reasonable method is to use equation 2.10, modified empirically by increases in numbers sampled in the larger or more variable primary units.

When the object of the sampling is to measure some total quantity, e.g. the total weight landed of a certain species of fish, rather than some mean value, e.g. the average length of fish, the analysis of the results, as given in equations (2.11) - (2.13) must be modified. The total in the i^{th} unit sampled will be

$$X_i = \frac{N_i}{n_i} \sum_{i=1}^{n_i} x_i$$

where $\frac{N_i}{n_i}$ is the raising or weighting factor for the i^{th} primary unit, and is equal to the reciprocal of the proportion sampled. The total in the whole population is given by

$$X = \frac{N}{N_i} \cdot X_i$$

where N = total number of individuals in the population. If N is not known, as may well happen, then the proper raising factor N/N_i cannot be used, and the approximation K/k has to be used, where K is the total number of primary units and k is the number of units sampled. (if the number of individuals in each primary unit is the same, the two raising factors will of course be equal). The use of the two raising factors in succession - from sampled individuals to whole primary unit, and from sampled primary unit to whole population - is most important. Serious bias can arise by using wrong weighting factors, if there are big differences in composition between primary units, especially if these are correlated with the number of individuals in the primary unit. Suppose for example we wish to estimate the total quantity of a certain species of fish living predominantly inshore, landed at a certain place. We may take as the primary unit the catch of a single boat

and sample from selected boats one box of fish. It is likely larger boats will work further offshore and have larger catches, and also have a smaller proportion of the inshore species in their catches. If the samples from these boats were only given the same weighting factor as those from the smaller inshore boats the proportion of the inshore species would be seriously overestimated.

Example 2.4.4.

At a certain landing place 30 boats landed fish. One box of fish was sampled from each of 10 boats, and the weight of two species of fish determined, with the following results:

Boat number	1	2	3	4	5	6	7	8	9	10
Number of boxed landed	28	10	16	20	18	12	10	5	15	25
Weight of species A in 1 box (kg)	10	1	2	2	7	8	3	2	9	12
Weight of species B in 1 box (kg)	1	10	2	2	2	7	3	9	8	2

Calculate the total weight of each species landed (a) using information given above, (b) using the additional information that the total landings by all boats was 450 boxes. Compare the ratio of the two species in the total landings, with the ratio in the 10 boxes actually sampled. (one box equals 50 kg)

3. SAMPLING THE CATCH

3.1. Sampling methods for catch and effort statistics

3.1.1. Statistics required

The first stage in the study of a fishery is to know what the fishery is catching. All too often catch statistics are non-existent, or lacking in essential information, e.g. division of the total catch by species or type of fish. Even more often there is no data on the fishing effort, even in the crudest form, e.g. number of boats, number of fishermen. Catch, effort, and their ratio, catch per unit effort, which gives the simplest index of the abundance of the stock, are the basis of the study of any fishery; once any two have been determined, the third can be at once calculated. However, the catch per unit effort need not necessarily be determined from the other two, but rather perhaps the catch and catch per unit effort determined directly, and effort duduced therefrom, or perhaps the catch deduced from data on effort and catch per unit effort. For this reason it is not possible to discuss sampling for catch or effort as separate problems.

Measures of fishing effort

A distinction must be made between the various uses of effort data and the units involved. An economist may regard effort as a measure of the economic input, and the catch per unit effort as a measure of the success or efficiency of the fishery. A biologist regards effort as a measure of the mortality caused by fishing, and the catch per unit effort as a measure of the abundance or density of the fish stock. Thus they may require different units of fishing effort, and in particular the units used will change with time in different ways. For example, suppose outboard motors are introduced into a fishery hitherto using only canoes with paddles. To an economist the input has changed little - the main factors, numbers of men or canoes remaining the same - and the catch per unit effort should increase. To the biologist the stock is unaltered and therefore the catch per unit effort should also be unchanged, but the fishing effort should rise. That is, the economist's unit of effort might be the canoe or fisherman, but the biologist requires units of unpowered canoes, a canoe with engine counting as perhaps two unpowered canoe units.

These complications, which tend to increase as a greater range of ancillary equipment and of modifications and improvements to the fishing gear come into general use, mean that fishing effort data is often most conveniently collected in two stages. First, a relatively simple measure of total effort, such as numbers of boats or number of voyages, is obtained perhaps by sampling, often closely linked with the collection of data on total catch. Then detailed information on, e.g. the size of boats, the specifications of their gear or other equipment, can be obtained from a section of the fishery to convert the simple measure of fishing effort into the more detailed measure required for the particular user. The detailed information may be collected from a sample of the fishery each year, but sampling can also be done on a time basis; for example I.C.N.A.F. collect detailed information (of size of ship, gear, etc) for all ships operating in the area, but only at intervals of several years.

3.1.2. Definition of population and choice of sampling unit

As in any sampling problem a first essential is to define the population being sampled and to choose an appropriate sampling unit. For this purpose it is nearly always easiest to consider where the fish are landed or first become available for weighing or recording, rather than the position where the actual fishing operations

take place. Most simply, the fishery may be based on a number of distinct landing places each of which can be considered as a unit. In a more complex situation landings may take place anywhere along a line - a coastline, or a river, and the natural unit is a certain length of coastline or river. The size of the unit is best taken sufficiently small for one man to cover, preferably in a day, but the limits may be extended when the fishing activity is very sparse. Finally, some, usually primitive, fisheries take place all over an area, e.g. in swamps, or irrigation channels - the fish either being consumed locally or sent to market in small batches. Here the unit may be an area of ground. These types of unit may well be combined, so that a stretch of coast could be divided into a certain of distinct landing places, and lengths of coast between these points .

The first step would be to achieve some stratification, dividing the units (landing places. stretches of coast) according to the order of magnitude of their fisheries. This needs some preliminary survey of the fishery, and, for the fisheries without definite landing places, possibly some geographical survey to delimit precisely the boundaries of the units. Where the division between units may be drawn rather arbitrarily on a stretch of fairly uniform coast it is best to draw the divisions to make the catches in units of the same class as equal as possible. This reduces the within-strata variance to a minimum, and hence increases the precision of any final estimate of total catch, etc. for a given amount of sampling. The division into say, large, medium and small may change through the year for a seasonal fishery, so that a landing place, or stretch of coast, classed as 'large' during the main fishing season might be classed as 'small' for the rest of the time.

The intensity of sampling would vary according to the importance of the unit. It may be possible to collect complete information from large units, but only sample 1% of the small units at one time.

Within a single population unit as so far defined, e.g. a single landing place, further stratifications may be desirable. As suggested earlier, one stratification might be by time. For instance, one length of coast may have been divided into eight units (e.g. landing places), and labour is available to sample about one-tenth of the time at each. Then each month's landings at each place could be treated as a separate strata, with landings for three days each month being sampled. The arrangements of these three days in the month would depend on such circumstances as the ease of travel between places. If this was good, so that there is no difficulty or great expense in recording at point A on Monday, point B on Tuesday, etc. then it would be better to space the three sampling days at roughly ten-day intervals through the month. (Travel between points would also be easier if landings only take place in the morning, leaving the afternoon for possible travel, rather than throughout the day). Conversely, if travel is difficult, the three sampling days may have to be taken together, even at the cost of some loss in precision due to the probable correlation between landings on successive days.

A second possible stratification, particularly at the large places, is by class of boat or gear. Even on a day when statistics are being recorded at a landing place it may not be possible to record details of all the landings, and details of catch and effort will have to be obtained from a sub-sample of the total. This will be more precise if different classes of boat (either in terms of size, or of type of gear used - lines, gill-net, etc) are sampled and analysed separately. This, of course, requires that some record of the total number (including those not sampled for catch, etc) of each category of boat landing on the day.

Ex. 3.1.1. Sixteen boats landed at a certain port; catches of two hand-liners and two gill-net boats were examined; they landed 45 kg, and 55 kg, and 75 kg and 105 kg of fish respectively. (i) What is the total estimated landings. (ii) If it was known that six boats used lines and ten gill-nets, what would be a better estimate 1120 kg or 1200 kg).

3.1.3 Estimation of total catch

The quantity of fish in a catch cannot normally be measured directly after they are caught, and the basic statistical data has to be collected at some intermediate point between the fisherman and the consumer. Usually this will be when the fish are landed, often at a market, where they are weighed and sold to, say, a wholesaler. These quantities will not necessarily coincide with the catch, any unmarketable fish (unpopular species, undersized fish, etc.), and also any fish kept by the fisherman for his own consumption being excluded. The possibility of such losses will always have to be examined, and if they might be significant they will have to be estimated by some such sampling scheme, as described below (3.1.4). Similarly, if the basic statistics are collected at any stage later than at first landing, intermediate losses have to be estimated.

For instance in Northern Rhodesia there is an important fishery on Lake Mweru, with its major market on the Copper Belt, some 250 miles away. Fish sent to this market have to pass a customs post, where a complete record of these exports can be made. Complete records of all fish landed are not so easy to collect, but it is possible, from sample observations, to establish what proportion of the total catch is exported, and hence estimate the total catch from the quantity exported.

Conversion factors will also have to be used if the fish are processed in any way before landing, so that the landed weight is different from the weight caught. The commonest example is, of course, gutting and cleaning the fish, where the conversion factor is small and easily determined, but the fish may also be dried or salted before reaching the market. An important special case of the use of such conversion factors is when nominal catch statistics are collected for purposes or agencies outside fisheries. These are often seriously incomplete, either because they only intended to include particular parts of the catch, e.g. that marketed rather than subsistence fishing, or because of deliberate falsification. A common cause of the latter is when returns from the fishermen are used for tax purposes. In this case the establishment of a reliable conversion factor (the proportion of the catch not reported to the authorities) may be a matter of some difficulty. It may be better for biological purposes to abandon all use of extensive nominal catch statistics and use completely independent estimates obtained from some sampling system.

Further sampling may be needed to determine the catches within the primary population unit - a landing place, or length of coastline - for the required interval of time - perhaps one day. Sometimes it will be possible to record all the catches being landed. For instance, all the catches may be laid out in boxes for sale by auction, and the quantities easily noted. At other places the sampling unit may cover a considerable length of shore, with perhaps several boats landing at once, and it is impossible to record details of all boats. Hence it will usually be that catches may be estimated from the total number of landings and the catch per

landing. This supposes that while recording details of one landing it is possible at least to note how many other boats are landing; in the extreme case even this is not possible, and another step used, such as estimating the number of landings per hour over part of the day only. Then the recorder would divide the day into one part (say n hours) recording the total number of landings (say N), and the rest of the day (m hours) on recording details of some of the landings (say k landings) in that period.

Then the average catch per landing $= \frac{1}{k} \sum_{i=1}^{k} w_i$ where w_i is the catch of the i^{th} landing whose catch is measured.

and total number of landings during day $= (m+n) \cdot \frac{N}{n}$

and the total catch $= \frac{1}{k} \sum w_i \cdot N \cdot \frac{m+n}{n}$

The allocation of time within the day between recording number of landings and catch per landing depends on the relative variance of the two quantities. Probably the catch per landing is the less variable, and will require less sampling. Landings per hour are more variable, and also are likely to vary systematically through the day. Thus, it is best to spread recording of landing rate throughout the day; a possible pattern is 1 ½ hours recording numbers of landings, and ½ hour recording quantity per landing repeated through the day.

Once an estimate of the catches at the landing places and days where observations were made, is available, the total catches at all landing places and days can be made in several ways. First, if no additional information is available, the sampled places may be assumed to be representative of all places, the recorded catches may be raised by a factor equal to $\frac{N.T}{n}$ when N is the total number of landing places, T is the number of days in the period considered, and n is the number of landing - days on which observations were made (n = m x t if observations were made at m places on each type of t days).

Example 3.1.2.

Sample observations were made at each of six landing places on a stretch of coast on four days during April, and the landings were:

(a) 350, 480, 320, 350 kg
(b) 180, 170, 250, 300 kg
(c) 280, 310, 200, 210 kg
(d) 370, 230, 250, 250 kg
(e) 280, 350, 370, 400 kg
(f) 400, 430, 380, 390 kg

What is the estimate of the total landings along the coast, for the month, given that there are in all 25 landing places:

$$\left(\frac{25 \times 30}{24} \times 7,500 = 234,375 \text{ kg} \right)$$

Often additional data available can be obtained on the relative importance of the landing places at which there has been no direct observation of the quantity landed.

For instance data on the number of fishermen, or of number of boats may be collected, either specifically as part of the collection of fishery statistics, or for some other reason, e.g. for tax purposes. Such data are likely to refer to a period, the duration as far as possible chosen to give reasonably constant conditions, perhaps for a year in a stable fishery, but monthly or even weekly if the fishery has considerable fluctuations.

Then the data for the landing places where the catches are observed directly will give an average catch per boat (or per man), and this average, multiplied by the total number of boats (or men) for the whole area gives the total catch.

Example 3.1.3.

In the example 3.1.2, the number of boats at the six sampled landing places was 20, 12, 15, 18, 20, and 25 respectively. At the other nineteen landing places there were, as counted at the beginning of the month, 250 boats. What is the estimation of the total catch, using the additional information on numbers of boats (catch per boat per day $= \frac{7,500}{110 \times 4} = 17.045$ kg, total catch $= 17.045 \times 360 \times 30 = 184,086$ kg).

Note that this is lower than the previous estimate, because the average number of boats at the sampled landing places is greater than elsewhere. This could well arise in practice when a proper system of random sampling is not used, as it may well be more convenient to base recorders at the larger landing places.

3.1.4. Estimation of quantity discarded

Statistics of catches and landings are often treated as being synonymous (apart from the need of converting landed weight of gutted or salted fish to weight as caught, of whole live fish). Often, particularly in Northern Europe and America, there is considerable rejection by fishermen at sea of both unwanted species and unwanted (usually small) specimens of commercially valuable species. Observations on shore, such as those described in the previous section, will only provide data on the quantity landed. For some purposes, particularly when assessing the effect that the fishery is having on the stock, data on catches are required. (Generally it may be assumed that fish caught but discarded will be dead, and therefore removed from the stock. If some fish may survive, then the significant quantity is the quantity of discarded fish that die; estimating the proportion of discards that die will require a further collection of data, e.g. by observers on commercial ships or special experiments).

Only someone at sea on the fishing vessel will be aware of how much fish is rejected, but generally fishermen will be too busy to keep regular records of the quantity rejected. Thus complete statistics of quantities rejected are very unlikely to be obtained, and if obtained are likely to be inaccurate. A sampling survey, either by sending special observers on fishing boats, or by obtaining special co-operation of a sample of the fishermen, is probably necessary if it is likely that any appreciable quantity is being rejected. Fishermen may well be prepared to keep accurate records of fish rejected on, say, one trip in twenty where they would not do so on every trip. Special observers sent out on board commercial vessels at sea, will secure accurate data on the quantity and type of fish rejected. However, they

will often be inefficient, in the sense that the ship they are on may continue to fish in one fishing ground for some time, with the same proportion being rejected, this proportion being different from that on other grounds. Thus the variance of the estimate of the average proportion rejected in the whole fishery will be rather high relative to the time spent recording rejection. If the special observer can also carry out additional observations, such as examination of the catch for size, maturity, feeding, etc, then wasteful repetition of similar records of quantity rejected may be eliminated, and the quantity rejected may be estimated accurately and efficiently.

The proportion rejected usually varies very markedly both with season and fishing ground, so that stratification is desirable. Within one strata, i.e. on the same ground during one season, the proportion rejected is likely to be fairly constant, though possibly varying from one year to the next, e.g. being large when a strong year-class is just below commercial size. Thus the accuracy obtained is likely to be high, even with very few samples - half-dozen samples within each strata should give estimates of serviceable accuracy. As usual the intensity of sampling should be governed by the expected quantity rejected. In the extreme case there may be certain seasons or grounds, particularly spawning grounds, which are known to have negligible, or no, rejection and thus may be omitted from the sampling system, permitting a greater intensity of sampling at other seasons or grounds.

Example 3.1.4.

During four voyages in the summer five tons of small fish were observed to be discarded, and forty tons landed; during a single voyage in the winter only 0.1 tons were discarded, and eleven tons landed. What is the estimated weight discarded by the whole fishery if the total landings were 4,000 tons in the summer and 5,500 tons in the winter (500 + 50 = 550 tons).

If the information is obtained from fishermen's records rather than by special observers, two procedures are possible: either to obtain a sample of fishermen who would give complete information from all, or most, of their trips, or to get information from all fishermen, but each one only providing information from a small fraction of his trips.

The first system, while probably much the easiest to operate, would have the drawback of being in no way a random sample, including only the more co-operative fishermen. For instance, it is likely that the fishermen who use small mesh or blinded nets (and thus catching a high proportion of small fish) would be less likely to be included. Thus the sampling would be biassed and the estimates of the proportion rejected would be too low. Before accepting as unbiassed the estimate obtained from the data received, it is important that some check be made on how the average rejection might deviate from that of the sampled boats, e g. by considering the abundance of the smallest sizes of fish landed, etc,

A rather similar problem to rejection is that of the processing, e.g. of fish meal at sea, where it is impossible to determine the composition of the catch by species by any examination when the catch is landed. Again, as in rejection, the more important problem is to determine the quantity of the small undersized specimens of the commercially valuable species, rather than of all species.

Data may be collected either by special observers, or by records of fishermen. The proportion of each important species may probably most conveniently be collected relative to the quantity processed (as this total will be known from landing records), rather than to the total catch.

3.1.5. Effort statistics

Simple effort statistics, for instance in the form of number of fishermen, or number of boats, are rather easy to collect, usually without a sampling system and in fact can be used (as in section 3.1.3.) as an intermediate stage in estimating the total catch. Such statistics, however, are unlikely to be good enough - i.e. closely enough related to the real intensity of fishing on the stock or stocks - for all biological purposes, particularly in any developing fishery. More sophisticated measures of fishing effort are likely to be needed, involving fairly precise measures of the time spent fishing, the size of boat, number or size of nets, etc.

The most suitable units of effort, in the biological sense, may not even be known at the time the statistical data are being collected. Thus, for a shoaling pelagic fish such as the anchovy, the catch per haul of a purse seine is probably, if corrected to a standard size of net, a good measure of the density, or size, of the individual shoal on which the net was shot. The actual abundance of the entire stock may, however, be more closely related to the number of shoals, or the distance between them, than to the size of the individual shoals, which may remain much the same size for a range of stock abundance. The number of shoals will be estimated from the time which has to be spent by the fishermen looking for fish before shooting the net. Corrections will have to be made for any devices which help the fishermen to find the fish more quickly, e.g. quicker vessels, radio communication between vessels, echo-sounders, etc. The resulting expression for the estimated index of stock density, in effect the catch per unit corrected effort, will be a possibly complicated combination of searching time and catch per haul, and one whose form may well be repeatedly revised on the basis of further research. However, the problem is a reasonably straightforward problem of fishery research, provided all the relevant data have been collected.

Most of the detailed information would be collected by some sampling method. Thus full information might be collected on the number of landings - though this itself might be obtained by another sampling system, as described for catch statistics earlier. From a proportion of the landings, say one landing in seven - perhaps from all boats landing on one day each week, fuller information is collected, say on total time away from harbour, divided into time steaming to the fishing grounds, time searching for fish, actual fishing time, and number of hauls made - perhaps distinguishing successful hauls, and those in which no fish were caught. Information could also be collected during these sample interviews on such things as size of net, radio and echo-sounding equipment, etc. These, however, will not change, except very occasionally, and more accurate information would be obtained by complete surveys, at intervals - possibly this would be combined with annual registration, etc.

3.2. Sampling for length composition

3.2.1 Methods of measuring and recording

Of all sampling work carried out by fishery research workers, the most extensive is almost certainly that concerned with determining the size and age composition of the catches. It is useful to distinguish two types of sampling programme (though as in most classifications the dividing line is not exact). First there is the thorough and precise examination and measurement, often of several characteristics (e.g. total length, fork length, weight, gonad weight, maturity age etc.), but of relatively few specimens. This sampling is usually done under laboratory conditions. Secondly there is the sampling for a single rapidly determined measure (e.g. total length), for a large number of specimens, often under the rather uncomfortable conditions of fish markets.

So far as the first type of sampling is directed at determining racial or specific differences e.g. in the ratio of two body dimensions, or in mean vertebra number, the sampling problem involved is small. The precision achieved is determined almost entirely by the number of specimens examined, regardless of the method of collection. Tests of significance between sets of samples will determine whether sufficiently large samples are being taken for the purposes required, either of showing that certain groups of fish are distinct or that possible differences within a group are less than a certain limit.

Slow and detailed sampling is also used to estimate directly the composition of the commercial catches by age, maturity, etc. This procedure will most often be a very inefficient sampling method, a much more preferable procedure being to relate these quantities to length by a relatively small key sample, and hence estimate their frequency in the catches as a whole from relatively extension length measurements. Most sampling is likely therefore to consist of numerous length measurements, for which suitable techniques both in statistical design and in manipulation, can achieve great improvements in efficiency. Though the manipulations concerned are also discussed in another manual in this series (another good discussion is given in the report of the F.A.O. training centre, 1958), it is appropriate to give some short discussion of the problem here. The main questions to be settled are, measure to be used (total length or fork length etc.), precision of recording (as millimetres or coarser grouping) and measuring device - (measuring board, etc.). As all measures of length are extremely closely related to each other, and to other measures of "size" (weight etc.), the one to be adopted should be that which permits the most rapid and accurate measurements to be made under the often rather difficult conditions of market and easily determined. The actual choice will depend on the species of fish concerned; most often the total length will be best, but if the tail has a well marked fork, length to the angle of the fork (fork - length) may be more suitable. A minor question is the use of fish which have been so damaged as to make the length used indeterminate, (e.g. badly frayed tail). These might be simply omitted from the sample, but it is not unlikely that damage correlation with size; bigger fish being more liable to damage.

O mission could therefore give a biassed result, towards underestimating the abundance of large fish, and a better procedure is to estimate the size, which is most readily done by comparison with undamaged fish of about the same size.

In an earlier section it was mentioned that in studying frequency distribution, of length or any other quantity, that undue precision - i.e. a large number of length-groups, - was not desirable, being extremely cumbersome to analyse. It has also been shown (e.g. Holt, 1958) that in attempting greater precision, the accuracy actually achieved can decrease; thus in measuring lengths to the millimetre, concentration on the last figure on the scale can cover the centimetres to be misread (183 mm read as 173). Similarly when plaice have been marked under rather difficult conditions it is not usual to find they have apparently grown (or shrunk) exactly 10 cm in a few days (though changes of say 7, or 13 cm are very rare). A rough guide 15-25 effective groups has been suggested (Yule and Kendall, 1950) with perhaps more where data is very abundant, and the frequency distribution complex. Thus the grouping suggested for Rastrelliger, whose lengths in the catches run from around 13 cm to 22 cm, is half-centimetres, giving 18 groups, while the standard for cod in the North-West Atlantic, which range from 40 to 130 cm is 3 cm, giving about 30 groups (this latter is perhaps too fine a grouping). The markings on the measuring board will be determined by the groupings used - if 1 centimetre groups are used, the 1 cm markings should be made clear and distinct, and not confused by the addition of 1/2 centimetre or millimetre markings.

The simplest method of recording is for one man to do all the handling and measuring of the fish, while a second man records the lengths on a suitable form. The design of this form will be discussed in more detail in a later section, when the additional information needed, other than purely the number at each length, will be discussed. In some forms the lengths are written down in serial order as the fish are measured, being re-tabulated as a frequency distribution later. This is convenient in detailed sampling, where several attributes - length, weight, maturity, etc - are being recorded for the same fish, but normally it is much better to record in a manner that a frequency distribution is immediately produced. This may be done by having a form printed with the possible range of lengths down one side. As each fish is measured a mark is made against the appropriate length on the form, and the number at each length can be simply totalled when the sample is finished. Totalling is easier if the marks are arranged in a system e.g. by grouping the marks in fives - four vertical and the fifth diagonally across them, or better, the first four as sides of a square, the fifth as the diagonal (or). As this requires two men at the sampling place, this system may be wasteful of man-power when sampling is done at a distance from headquarters, though the second man is often useful in helping move boxes of fish, etc. A direct modification is for the measurer to talk into a small tape-recorder; the tape can then be played back when convenient and the measuring form made out in the normal fashion. Though the actual time involved is the same, the form can be made out at any time, rather than very early in the morning. This system is being used in several European countries.

A simpler method for one man to produce a permanent record is the use of plastic or celluloid strips. These are placed on the board, one strip being used for each sample. No specific measurements are made while handling the fish, but a hole is punched in the strip at the end of the tail (or for whatever measure of length is being used). On return to the laboratory the strip is replaced on a measuring board, and the number of fish at in length interval is at once given by the number of holes in that interval. A somewhat similar system is to use a type of plastic on which pencil mark will not rub off too easily. The measuring board is covered with this plastic, and the length intervals marked permanently on it. A pencil mark is made in the interval in which the end of the tail (or fork if appropriate) falls, grouping into fives as before if necessary. This, like the punching of the hole for the previous method, can be done with very little delay in handling the fish. At the end of each sample the numbers in each group can be totalled, and entered on the suitable form.

3.2.2. Selection of the sample

When the fish are lying in a large heap, perhaps of several hundreds, or even more - for instance herring on the deck of a research vessel, - from which only a proportion are taken and measured, the wrong method of selection can give rise to serious bias. Most people tend to take first the larger and more obvious fish. A sample taken without care is therefore likely to be seriously biased; a nominally "random" sample of cod taken for age determination on board the English research vessel ERNEST HOLT had an average length more than 10 cm greater than catch as a whole. Even if the sampler is aware of this bias, it cannot be eliminated with certainty, and the chance of over correction - picking too many small fish - is introduced. The danger of bias is most directly eliminated by measuring all the fish in the heap, and if this is reasonably practicable it should be done, even if the sample size is rather larger than would otherwise be desirable. If the complete heap is too large, the number to be measured may be reduced by measuring only a part, e.g. one-fifth of the total. This may be done by picking up all the fish, and separating them into 5 lots e.g. 5 baskets, one of which is then chosen at random and the fish in it measured. The fish should be allocated to each group a few at a time, so that each group contains some of the fish picked up first (probably larger than average), and some of those picked up last (smaller than average). It is probable, though, that some groups will still have more than their fair share of big fish, and improved degree of mixing may be achieved by repeating the process. That is, all the fish can be re-allocated, a few at a time, to 5 new groups.

Alternatively the sample to be measured may be determined by its position, all the fish on one side being sampled. It is important that all the fish in the section are measured, the area sampled being cleared right down to deck level. Bias is still possible, the larger fish being more likely to be in one part of the catch than another, and this can in part be corrected by sampling different parts of the catch at different times; one haul the after end is sampled, the next haul the forward end.

Commercial catches of any quantity are normally placed in boxes, baskets or similar containers, and often one or more complete boxes will form a convenient size of sample. Here there is no risk of bias through picking up the larger fish first. If a complete box is too large a sample and only part of the fish are measured then precaution must be taken against this bias. As before this may be done either by handling all the fish, separating them, a few at a time, into two or more groups, or by taking all the fish at, say, the left hand side of the box, from the top right down to the bottom.

3.2.3. The time and place to sample commercial catches

Great care is necessary to ensure that the system used in choosing the time and place to take samples will give unbiased estimates of the commercial catches. Large systematic differences are likely to exist not only between fish caught at different places or by different gears, but also, after landing, between fish sent to different markets, or used for different types of processing. Most biologists are fully aware of the former chances of bias, but the latter may well be unsuspected, and give rise to serious bias.

For instance, an apparently convenient method of obtaining a sample is to arrange for a particular fish merchant to send a "random sample" of fish to the laboratory. This merchant may be accustomed to buying a particular size or condition of fish for his own market, e.g. large fish so that the sample would be biased towards the larger sizes, while there is the added risk that the fish sent to the laboratory will be the cheapest size or condition. Sampling, therefore, should be carried out as soon after the fish are caught as possible, before any bias is introduced which cannot be corrected. This of course also means that it will be easier to determine where, when, and by what gear the catch has been made.

Ideally, then, sampling of commercial catches will be done actually on board during the fishing operation. It may be difficult in practice to send scientists on board, and often the method will be inefficient. That is the scientist will have to remain on board the whole trip, perhaps several days, during which fishing is only done on one or two grounds, so that in a week he may only effectively take a couple of samples. In rather less time a scientist ashore can take samples from perhaps twenty or thirty boats as they land, getting information from a corresponding variety of grounds, and, perhaps, gears. This inefficiency will be greatly reduced if the scientist can also take other observations on the research vessel, e.g. on the quantities of small and unmarketable fish rejected, the feeding of the fish, possibly weather and hydrographic observations, etc. While such observations are not strictly part of the sampling problem discussed here they may be integral parts of a complete research programme, consideration of them is vital in determining whether or not sending observers on commercial vessels is worth while in terms of economical use of man-power.

If observers can transfer at sea from one ship to another, then the number of samples that can be taken per month, or per day will be increased, and with them

the general efficiency of the sampling method. For instance it would be possible in the purse-seine fishery of the Gulf of Scam, for observers in a small boat to go among the fleet each night, board a vessel when it has made a catch, and measure a sample of fish noting the time, vessel and exact place of capture, all in a very short time. This could be repeated several times, so that perhaps a dozen samples, from accurately known positions and times, could be made in a single night.

For most fisheries the most suitable opportunity for sampling is directly after the fish are landed. Some markets e.g. in England or Japan represent the most favourable conditions. Here the fish are landed during the night and are sold by auction in the morning. There may be an interval of an hour or more after the complete catch has been landed and laid out on the market when the men landing the fish have left, and before the majority of the fish merchants have arrived for the sale of fish during which the catch can be very easily sampled without disturbance between the scientists and the fish merchants. In such conditions the techniques of measuring, in terms of time and space needed, are not critical, but elsewhere both in England and other countries, the period of calm may be small or non-existent, the fish being landed and sold without delay. Even so there is nearly always a few minutes during which any particular basket of fish is available for sampling, provided the scientists are able to work quickly and in distinctly difficult conditions, and provided there are reasonably good relations between the scientists and the other people on the market. These last two requirements may sometimes be conveniently met by a small payment to the fisherman for measuring his catch. This is likely to be most suitable in a fishery consisting of a large number of small fishermen each landing only a small catch, and for which a convenient sample might be the entire catch of one fishermen.

Sampling at the place of first landing may be difficult or impossible, for instance because the necessary staff cannot get to the landing place at the right time. Sampling will therefore have to be at the first convenient stage in the chain of distribution, e.g. at the wholesale market. The further the sampling is removed from the actual time of capture, the greater is the difficulty in determining the details of capture (especially the position and gear) so that valuable biological information is likely to be lost. If all the catch does not go through the same channels - for instance some goes to the wholesale markets for sale as fresh fish but the rest is sent for salting, - and regular sampling can only be conveniently done at say the wholesale market, then the fish sent for salting have no chance of appearing in the samples. Thus the sampling system cannot give a random sample of the catch, and periodic checks must be made of the composition of that part of the landings which are not regularly sampled to ensure that they do not differ from the rest of the landings. For instance fish used for the production of fish sauce are likely to be smaller than those used as whole fish. If such differences do exist unbiassed estimates of the catches can only be obtained by sampling all the channels of distribution or, better, overcoming the difficulties and sampling the fish when first landed.

3.2.4. Stratification

An ultimate object of sampling may be to determine the composition of the total catches of a certain species for a complete year. This is too large a unit to be sampled as a whole, and some degree of division, or stratification is necessary for practical reasons, as well as to increase the efficiency of sampling. Likely divisions are by season or time of landing, place of landing, or method of capture. Thus a particular division may be the landings during August, on a particular stretch of coast, by vessels using gill-nets. From all the fish landed at this time and place one or more samples will be taken, and the composition of these landings estimated; similarly the landings at other times, at other places and by other gears (if any) are sampled and their composition estimated, and hence by addition the composition of the total landings obtained. A vital part in doing this is the knowledge of the magnitude of the catches in each division, so that the correct "raising factors" can be used. The raising factor is the factor by which the numbers in the sample have to be multiplied to give the total numbers in the population sampled.

Example 3.2.1.

Samples of Hilsa, each of 50 kg, were taken during 4 winter months. The length composition of each sample is given below:-

Length (mm)	Dec.	Jan.	Jan.	Feb.	Mar.
255-	18	7	4	2	3
275-	16	9	6	4	2
295-	18	11	12	3	4
315-	10	14	16	9	6
335-	7	10	19	12	11
355-	6	9	15	16	10
375-	5	12	11	8	14
395-		6	5	12	8
415-			8	7	4
435-				3	2
455-					6
Total	80	78	96	76	70

(The notation 255-used for the length groups means that the group includes fish of from 255 mm inclusive up to, but excluding, 275 mm).

The total weight landed in each month was 250 kg in December, 1000 in January, 2500 in February and 500 in March.

Determine the total numbers landed in each length group, and also the percentage length composition of the total landings. Compare the latter with the percentage length composition of the 400 fish measured.

The raising factors for each month are 250/50 = 5, 1000/100 = 10, 2500/50 = 50 and 500/50 = 10. (Note that two samples were taken in January, so they are added together before raising; the raising factor is, of course, half what it would be if only one sample was taken). Using the appropriate raising factor for each month and adding up, the length composition of the total landings is obtained, as follows:-

	Numbers landed						Numbers measured	
Length	Dec.	Jan.	Feb.	Mar.	Total	%	Total	%
255	90	110	100	30	330	5	34	8.5
275	80	150	200	20	450	7	37	9
295	90	230	150	40	510	8	48	12
315	50	300	450	60	860	13	55	14
335	35	290	600	110	1035	16	59	15
355	30	240	800	100	1170	18	56	14
375	25	230	400	140	795	12	50	12.5
395		110	600	80	790	12	31	8
415		80	350	40	470	7	19	5
435			150	20	170	3	5	1
455				60	60	1	6	1.5
Total	400	1740	3800	700	6640		400	

It may be noticed that the smaller fish are considerably over-represented in the samples. This is because the sampling is spread fairly evenly through the season, while the landings are greatest in February, when the larger fish predominate.

In calculating the raising factors the quantity being estimated must be clearly kept in mind. In the example above the quantity to be estimated is the numbers landed in each size group, and the raising factors are determined by the quantity landed each month. Quite different raising factors will be used if estimates are to be made of the average size-composition of the stock over the period considered. The raising factors for each month will be given by the size of the population, which will have to be estimated in some way.

Example 3.2.2.

If, in example 3.2.1. it is assumed that the sizes of the population in December, January, February and March were proportional to 2:3:3:2 respectively, calculate the average length composition of the stock.

The main problem to be faced in the stratification of the landings is the amount of sub-division to be done. For the application of the usual methods of stratified sampling, the division must be few enough for samples to be taken from each division; small and numerous divisions are desirable if there are great differences between divisions, and large and few divisions if the composition of possible divisions are similar each division must be small enough for it to be possible to take samples from it which are truly random. The number of samples taken from any division should be, as far as possible, proportional to the variance in that division.

There is no need for all divisions to be of the same size. Consider divisions in time, for example in a fishery which though continuing throughout the year, has peak landings in a short season. In this main season a week may be considered as a suitable division, samples being taken say daily, and in the off-season each month may be considered as a unit, with samples taken weekly.

Example 3.2.3.

The monthly catches of mullet (Mugil spp) from Lake Menzala, Egypt, in 1956-7 were:

Month	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Total
Catch (tons)	136	166	343	372	523	395	264	235	140	98	117	75	2864

If time and labour is available to take 10 samples during the year - suggest how they should be allocated.

It is not possible to say that any particular allocation is the "right" one, but a reasonable one would be to have five divisions, with two samples taken in each period; without knowledge of the size distribution in each month, it may be assumed that the variance of the length distribution is the same for each period - so that the weight landed in each period should be as far as possible the same, i.e., close to the average of 580 tons. If the sampling period starts in January, a possible division would be January-April; May-July: August-September; October; November-December.

Suppose that in a fishery with a similar seasonal distribution of total catch, it was known from earlier sampling that the main size groups caught in each month were as follows:-

Month	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
Sizes	19-21	20-22	21-24	22-25	23-25	24-27	20-28	20-28	15-28	15-18	15-18	16-20

How would this additional information alter the division into periods suggested above ?

It appears that for most months only one major size-group is present, except in the period December-February; most intensive sampling is necessary in this period. The rate of change of size is slowest in the peak fishing season (September-October) so relatively less sampling is necessary then. A suggested division into five periods is therefore January-February; March-May; June-September; October-November; December.

When dividing the total landings into ports or stretches of coast, the physical ease of sampling is important. The laboratory, or normal place of work of those who will do the sampling, is likely to be at or near an important landing place. Sampling at this place can therefore be done with little waste of time; perhaps two hours work actually sampling can be done at the expense of three hours away from the laboratory. Sampling elsewhere may mean the loss of a complete day's work or more. Information must, however, be obtained at these more distant landing places if a reliable estimate of the composition of the total landings is to be obtained. In some circumstances it may be possible to confine research to the fish landed at the most convenient place, regarding the stock of fish represented in these landings as a sample of the stock or stocks of fish exploited by the fishermen of the whole area. This may well occur when investigating an inshore species, where each stretch of coast may contain separate and discrete stocks. Reliable results are not likely to be obtained if the stock is exploited by various groups of fishermen and samples are only obtained from one group. Before arranging a complete regular sampling programme for the whole region, a preliminary survey should be made of the composition of the catches at each place. It may happen that there is no appreciable difference from place to place and regular sampling can be carried out where most convenient, subject to occasional checks elsewhere to make sure that the composition is indeed the same as at the main sampling area. A special case of this may occur in off-shore fisheries. The total landings at two ports may differ in composition because the main fishing areas of the two fleets differ, but total areas covered overlap. Thus the landings may be divided according to area fished, and all samples taken at one port, but weighted according to the total landings from each area to give unbiassed estimates of the total landings. Again, periodic checks are necessary to confirm that the catches from any particular area made by fishermen from different landing places do have the same composition.

Often, though, there is no alternative to including all major landing places in the regular sampling programme. The division of the total into areas, and the choice of size of sample taken from each division is then carried out in the normal

manner with one modification. Normally the best size of sample taken from any division is proportional to the size and variability within that division. This is true so long as the cost (in time and labour) of taking a sample of a given size is the same for all divisions (in this case, areas). If in fact the cost varies from area to area, then this has to be taken into account, more samples being taken in the areas where sampling is easiest and cheapest. In fact, the best size of sample in any area will be inversely proportional to the cost of taking a sample of unit size in that area. In mathematical terms the best size of sample, will be given by:

$$n \propto Ns^2 \quad \text{(equal costs in all divisions)}$$

and $$n \propto Ns^2/C \quad \text{(variable costs)}$$

where C is the cost of a sample of unit size.

As before it is not essential to obtain samples of exactly the right size, but it is important that cost is taken into account, and also that all sampling is not done in the easiest place.

Stratification, or division, by method of capture is often most important, both in understanding the fishery and in obtaining the best estimate. There is often a considerable difference in sizes of fish caught by different gears, making division desirable. Also, one gear may take a wider range of sizes than the other, thus requiring more intensive sampling.

Example 3.2.4.

The percentage length-composition of cod landed at Grimsby in 1957 caught by trawl and Danish seine respectively were:

Length (cm)	30-39	40-49	50-59	60-69	70-79	80-89	90-99	100-109	110+
Trawl	26.3	38.8	19.0	7.9	3.7	1.9	1.7	0.6	0.1
Seine	0.6	13.3	16.0	8.3	6.8	15.0	28.0	10.5	1.5

The total landings were 4, 500 tons by trawlers and 3, 500 tons by seiners. Calculate the variance of the mean length of fish caught by each type of gear, and discuss the allocation of sampling effort between the two gears:

Trawl: mean = 49 cm variance = 194 cm^2
Seine: mean = 78 cm variance = 440 cm^2

The size of sample giving the best estimate of the total mean length from the two gears, as judged by the weight landed, and the variance of the mean lengths should be in the ratio 45 x 194 : 35 x 440, i.e. about 1 : 1.7.

If in fact the samples taken were of 1500 fish weighing 1.9 tons from trawl-caught fish, and 500 fish weighing 2.2 tons. (These samples, by weight, are in the ratio 1.9: 2.2 = 1 : 1.2, which is reasonably close to the 'best' distribution, though rather more seine-caught samples would be desirable), calculate the total numbers at length sampled from trawl and seine, and hence, using the correct raising factor, the total numbers landed in each 10-cm. group.

The greatest benefit from stratification can be obtained when the fish are divided into different size categories. At the extreme, where such sorting is very detailed and consistent, information on the total quantities landed in each category will be almost sufficient to determine the total size composition. However, advantages from sorting into size categories can still be gained whenever the total landings are divided into classes which differ markedly in size-composition, even when the division is not very precise. So long as the total landings can be divided at all into, say two categories, 'large' and 'small', then the condition that stratified sampling is advantageous is satisfied - there is a bigger (probably much bigger) difference in size composition between categories than there is within a category.

Of course, separate sampling of each category must always be done when sampling the fish landed by one ship, but there is a question whether or not to consider categories when estimating the composition of the total landings. The two procedures may be illustrated by an example.

Example 3.2.5.

Plaice landed at Lowestoft are normally sorted into four categories - large, medium, small and thin, the last being poor quality fish of lesser market value. During a certain period the total quantities of fish landed were: Large 750 stones, Medium 1,500, Small 3,840, and Thin 110 stones - a total of 6,200 stones. A certain ship landed 30, 100, 480, and 10 stones of each category and of these 10, 5, 3, and 10 stones were measured, giving the following length composition:

Length Group (cm)	25-29	30-34	35-39	40-44	45-49	50+	Total
Large				16	33	15	64
Medium		2	47	27			76
Small	26	41	8				75
Thin			7	15	14	14	50

(i) Estimate the numbers in each length-group in each category landed by the sampled ship.

(ii) Estimate the total numbers landed in each length-group, (a) by raising each category separately to the total landings (stratified), or (b) raising the total landings of the sampled ship, of all categories.

(iii) Comment on the relative numbers sampled in each category.

The raising factors for estimating the total landings by the sampled ship are, for each category, 3, 20, 160 and 1 respectively, giving the estimated size composition, the total being obtained by addition.

Length Group	25-29	30-34	35-39	40-44	45-49	50+	Total
Large				48	99	45	192
Medium		40	940	540			1520
Small	4160	6560	1280				12000
Thin			7	15	14	14	50
Total	4160	6600	2227	603	113	59	13762

To estimate the total landings by all ships, using the data on the quantity landed of each category, the raising factors of 25, 15, 8 and 11 for each category respectively are used.

Length Group	25-29	30-34	35-39	40-44	45-49	50+	Total
Large				1200	2475	1125	4800
Medium		600	14100	8100			22800
Small	33280	52480	10240				96000
Thin			17	165	154	154	550
Total	33280	53080	24417	9465	2629	1279	124150

Another estimate can be obtained by raising the total of the sampled ship by the factor 6200/620 = 10, giving the estimate

Total 41,600 66,000 22,270 6,030 1,130 590 137,620

The second estimate gives more small fish (under 35 cm.) and less (and for some sizes considerably less) large fish. This is because the sampled ship landed proportionately more of the category small compared with the total landings, though neither estimate can be considered as definitely right (or wrong), the first one, which takes into account the quantity of each category landed by all ships is likely to be closer to the actual value.

3.2.5. Size of sample

The basic sampling unit is likely to be the catch (or more usually landings) by one ship or one unit of gear. The choice in size of sample therefore lies between measuring a large number of fish from one ship, and a smaller number from several ships. Sometimes the practical situation will limit the size of sample; if the fish are being sold, or moved away almost as soon as they are landed, it may be possible for a single box to be measured without disturbance where a larger sample would cause undue interruption to the fishermen's activities. This underlines the fact that good market sampling - and hence proper estimates of what sizes of fish are landed, and are in the sea - depend as much on the ability of the men doing the measuring (either singly, using "one-man" boards, or in pairs, one measuring, one recording) to work in often difficult conditions and maintain friendly relations with the fishermen, as on the statistical design of the sampling system.

Assuming a choice in size of sample, this will as usual be governed by the variances between and within units (ships, etc.) and by the time spent measuring fish and moving between ships. The variance between ships is usually less than the variance within ships, especially when all the fish landed by a particular ship come from the same ground, or have been caught in the same haul (e.g. of a purse-seine). Movement from one ship to another to take samples is likely to be

easy when catches are landed at the same time - e.g. just before the morning's auction sales; in these conditions small samples, and many of them, will be the best system. At other places ships may land at intervals during the day, so that a second ship's fish may not be available for sampling directly after finishing measuring from the first ship. Here larger, but fewer, samples will be better.

In example 2.4.3, data of some herring measurements were examined in some detail, and it was shown that, for a given time spent sampling, the mean length of herring was estimated with least variance if samples of 17 fish were taken. Before taking this analysis, or similar analyses for other situations, as measures of the best size of sample, two points must be noted. First, and most important, is that with such a small sample size, the problem of unconscious selection by the man measuring the fish may become critical (cf 3.2.2). If, as is usual, the fish are landed in baskets or boxes, a sample size of a complete box, or boxes, is best, unless this is much bigger than the indicated optimum size. In the latter situation at least enough fish should be measured to reduce the possibility of bias - e.g. half a box, measuring all the fish from one side. Secondly, length measurements are used and interpreted in many ways, and it does not follow that the sampling system which is most efficient in estimating the mean length of fish landed is also the most efficient in estimating other quantities, e.g. proportion of fish above a given size, or, in combination with a fixed amount of age data, mortality. However, the optimum size of sample is not likely to be very different, and mean length and its variance, being about the easiest to calculate, gives as good a guide as any.

Example 3.2.6.

At a certain market it takes half a minute to measure a shark; ships land their catches close together, so that it takes only a minute between finishing measuring the catch of one ship and starting measuring on the next. Assuming the same ratio of variances between and within ships as in Example 2.4.1, what is the nominal optimum sample size (3 fish).

How would the optimum sample size in both examples be changed if landings were infrequent, so that to sample a ship's landing required a special visit to the fish market, taking approximately one hour (60 fish, 20 fish).

So far the choice of sample size has only been considered for the simple situation where the landings by one ship are essentially homogeneous. Some slight modifications are needed if the landings have been sorted into two or more categories, based on size or condition of fish.

If the sorting is consistent - the same categories used for each landing - then each category could be treated entirely independently. For each category there is an optimum size of sample, and the division of the total sampling effort between categories will be made on the usual criteria of stratified sampling - more samples from the bigger or more variable strata. This will usually mean that the optimum number of samples is different for different categories, and

while there is no overriding necessity to do so, it is more convenient when sampling from a ship to sample from all categories, i.e. the number of samples is the same for all categories. This will mean some compromise is necessary, so that for categories where an above average number of samples would have been desirable, the size of sample is increased (e.g. instead of measuring 1 box of "large" fish, 2 boxes are measured). It may be emphasized that the fact that the optimum size of sample is small implies nothing about how much sampling is desirable, but merely that the sampling effort is best directed towards increasing the number of samples.

3.2.6. Raising factors

The use of raising factors has been mentioned in passing in earlier sections. In one form or other they are a vital step in combining and analysing sample data, and one whose importance is easily overlooked. Unless the sampling system used is very simple, the length distribution of the fish are measured will be different, and perhaps markedly different, from the estimated length distribution of the "population" being sampled - the population in this sense being possibly all fish of the given species landed during the year, or perhaps all those above a given size in the area being fished. When more than one sample has been taken from a population and each giving an estimate of the size distribution of the population, the estimate obtained from the combined samples may depend as much on the raising, or weighting, factors used as on the actual distributions of each sample. An extreme example is when the fish are sorted into fairly precise size categories. Then the most important information about the size distribution of fish in the landing as a whole is the quantities landed of each category. More than one set of raising factors may be used, each set being followed by a summation of separate individual raised samples or groups of samples; such a series of operations might be :

1. Measure a box of fish from a particular boat at a particular landing place, in a particular month.

2. Raise the sample to the boat's catch (i.e. multiply by a factor equal to the number of boxes that ship landed).

3. Add together all samples taken in that landing place in that month.

4. Raise to landings at given landing places for the month (i.e. multiply by a factor equal to the ratio of total landings by sampled landings).

5. Add together results for all landing places where samples were taken.

6. Raise to month's landings at all places (i.e. multiply by a factor equal to the ratio of total landings by landings at sampled places).

7. Add together results for all months to give landings for year.

Additional operations may be added to this sequence. Perhaps samples were only taken every other month; then the sum for the sampled months would be raised to give the year's landings, using the ratio of years landings by landings in sampled months. At the other end of the sequence perhaps only part of a box is measured, and the initial operation might be to raise this sample to give the estimated contents of a complete box.

Example 3.2.7.

Samples of Tilapia caught by ring net on Lake Nyasa were measured during December 1945 and January 1946. Details are given in the following table (data adapted from Lowe, 1952).

Month	Body length (cm)																					Total
	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
Dec.					1			3	4	8	10	28	25	40	34	32	18	4	1	1			209
Dec.							1	1	2	2	1	7	10	10	14	28	23	12	4	7	4	5	131
Dec.					2	2	2	4	3	6	4	25	28	24	26	22	11	5					164
Jan.		1	3	4	1	3	6	6	8	15	15	22	30	25	27	20	18	9	4		1		218
Jan.					1		3	6	6	8	6	14	23	19	19	23	10	8	2	2			150

The first two samples were the complete catch of single hauls; the third and fifth samples were one-half of the catch, and the fourth sample was one-third of the catch of one haul.

(i) Raise, where necessary, sample to give total catch of sampled haul.

(ii) Treating each month separately, raise sampled hauls to give total catch of each length for the month, and total for the two months, given that the catches in December and January were 118, 000 and 120, 000 fish respectively. (Note: raising factor for December= 118, 000/(209 + 131 + 328))

(iii) Calculate within haul and between haul variance of mean length for December.

The large between haul variance in this example suggests that the best sampling system would be to take many small samples and that the sample size here (150-200 fish) might be too large. However, the sample technique used was to send observers on the fishing boats, and to measure the fish directly they were caught (which of course is the best system to avoid any sort of bias in fishermen sorting the fish for sale). This means that once the fish from one haul were measured no other group of fish was available to measure, i.e. essentially the time between samples was large. No advantage to the measuring programme itself would be gained by reducing the size of sample, but of course there is always the

possibility in this situation of the observers doing other work - measuring fish of other species, collecting data on maturity, feeding, etc.

Two raising factors were used in the example above; the second - going from the sum of the sampled catches to the whole month's catch, is based on normal statistics of catch which will presumably have been collected independently of the sampling programme. The first factor, used to estimate the complete catch in the individual sampled hauls (used in the last three hauls), requires information which is only available at the time of sampling. Only if it is noted at the time whether the sample measured consists of all, a half or one third, etc., of the complete catch can the individual samples be combined with their proper weights to give the best estimate of the composition of the total catch. This is a general feature of nearly all sampling, that some information on quantities has to be collected at the time of measuring so that the correct raising factors can be calculated. The data required is the weight sampled and the weight of the catch from which the sample comes. Sometimes, when only part of a box of fish has been measured (because a complete box would be unnecessarily large), the fish remaining in the box which have not been measured will have to be counted to determine the proportion which has been measured. For instance a ship may land 18 boxes of whiting; 100 fish are measured, and the rest from one box (125) counted. The raising factor is then:

$$18 \times \frac{225}{100} = 40.5$$

Where many samples are regularly taken, much work can be saved by suitable lay-out of the computations. If the measurements have been recorded say in centimetres, but ultimately the data is required in wider groupings, say 3 cm groups, then the work is reduced by making the grouping as early as possible, thus reducing the number of size groups for which computation is needed. Thus the data in example 3.2.7. might well be grouped into 2 cm groups; and laid out as below.

Sample	Length groups												
	12-	14-	16-	18-	20-	22-	24-	26-	28-	30-	32-	Total	Raising factor
1			1	3	12	38	65	66	22	2		209	1
2				2	4	8	20	42	35	11	9	131	1
3			4	6	9	29	52	48	16			164	2
4	1	7	4	12	23	37	55	47	27	4	1	218	3
5			1	9	14	20	42	42	18	4		150	2
Total	3	21	23	71	131	255	438	429	206	33	12	1622	

The appropriate raising factors have been noted at the end of each row, so that (if samples from both months are taken together), the estimated number of fish in each size group caught in the sampled hauls is given by the sums of products of two columns - a procedure which is easily carried out on most calculating machines.

3.3 Indirect sampling

Fishery biologists are concerned with a wide range of attributes of individual fish other than length - e.g. age, maturity, sex, meristic characters, etc. Direct sampling for these can of course be carried out in a similar way as length sampling. However, the work involved for each fish is much greater and time consuming than length measuring, and cannot easily be carried out on normal commercial landings, but require the fish to be brought back to the laboratory. There are exceptions; for instance, the sex of some species (e.g. plaice) can be determined very quickly - in suitable light conditions it can be seen as the fish are picked up to be placed on the measuring board. Thus, provided the recording form is suitably divided, separate length data on males and females can be obtained virtually as quickly as length data with no regard of sex. In some fisheries, e.g. redfish, plaice, characteristics of growth etc. differ enough between males and females to make it desirable to treat them essentially as two different species, with length data, age-composition, maturities, etc., all computed separately. Nearly the exact converse occurs in some tropical fisheries. Here several very closely related species may be caught at the same time. Measurements on the market can only be referred to the species group as a whole, and separation into species only achieved in detailed laboratory examination.

When sampling directly for age, etc., the same principles apply in choosing the sampling system as when sampling for length. Bias is avoided by sampling as soon after the fish are caught as possible, before any selection for various market conditions is made, (or if this cannot be done, at least sampling from all market selections, and weighting the samples correctly). Variance is reduced by taking the right size of sample - which, because of the long time in taking the observations, and hence the large within-sampling time, is likely to be fairly small - and by choosing suitable stratifications.

In practice the need to avoid unconscious bias in selecting fish to sample will require the actual size of sample to be considerably larger than that required to give the least variance. Thus the smallest unbiassed sample is likely to be a complete box or basket (or half a box if the sampling is done carefully, taking all the fish from one side), or if the fish are not landed in containers, perhaps as many as 100 fish. This size is usually large, but not unreasonably large for efficient sampling with a quick operation such as measuring total length, but will involve quite appreciable inefficiency (in the sense of increased variance) with the longer operations such as age-sampling. However, if direct age-sampling is done, such inefficiency must be accepted, rather than introduce the risk of bias, which cannot be detected or corrected in subsequent analysis, and which could destroy the validity of most results.

Length sampling is essentially a very quick operation, and will normally be done at sea, or on a fish market during normal commercial operations, the fish once measured going back among the rest of the catch. Other observations, being long, and often involving mutilation of the fish, frequently require the fish to be bought by the research organisation. Rather than buy the fish from

the market, it may be possible to obtain the fish direct from the fisherman; ideally a sample would be put on one side during the actual fishing operation, with perhaps several such samples if the ship is, for example, a trawler fishing for several days on possibly three or four grounds. Such samples from a known position at a known time are potentially a great improvement on samples taken at the time of landing from fish possibly caught on several grounds. However, all will depend on the reliability of the fisherman, in putting the sample on one side at the time and place reported, and more particularly, on taking an unbiassed sample of the catch. The difficulty in taking a relatively small unbiassed sample from a catch on a fishing vessel's deck, even when there is no conscious selection, has already been mentioned (sec. 3.1). In addition there will always be the possibility, especially when fish are scarce and prices high, of the fisherman tending to put the poorer quality and less valuable fish into the scientist's sample.

Because, like many sources of bias, such selection is difficult to detect, it is often better to sample from the landings, where the entire sampling procedure can be controlled directly. An alternative would of course be to send a special observer to sea to collect the desired samples. The use of such special observers has already been discussed in connection with determining discards (sec. 3.1.4.), and they are very inefficient in terms of time spent unless a wide range of information is being collected - i.e. the actual time spent collecting any one item is small.

The normal procedure in sampling for age, maturity, etc., is therefore for a moderate-sized sample - say 100 fish - to be brought from the fish market to the laboratory; again, to avoid bias, it is better for fish to be bought by a scientist who actually goes down to the market, and best for the box actually taken to be selected by some random sampling procedure rather than sent up by a co-operative fisherman or merchant. Because of the rather high overhead costs of each sample in terms of value of the fish, time spent fetching them and finding out where they were caught, it is more efficient to make more than one type of observation. Thus the English herring sampling programme, besides a large number of measurements on the market, consists of a number of laboratory samples. Each laboratory sample consists of 100 fish, each of which is weighed and measured, the scales are taken for age-determination, maturity is determined and vertebrae are counted.

Many of the characteristics considered - e.g. age, weight, maturity, etc. - are closely related to the length of fish. Their distribution in the catches may therefore often be most easily estimated from a fairly large series of length measurements, together with a relatively small number of samples to give the relation between length and, say, age. In this way the total numbers in each length group in the catches (as estimated from the length measurements) are divided between the various ages in the same proportion as these ages appear in the key samples taken of that length-group. Essentially each length-group is treated separately.

Example 3.3.1.(adapted from Fridriksson, 1933).

A sample of 100 cod between 50 and 100 cm were measured and their otoliths used to determine their ages. Fish under 9 years and over 11 years old were grouped together. The results are given in the following table.

	Age (years)					
Length (cm)	-8	9	10	11	12+	Total
50-	3	1				4
60-	3	10				13
70-	7	32	1	7		47
80-	1	8	3	6		18
90-		3	3	5	2	13
100-				1	1	2
Total	14	54	7	19	3	97

A separate series of length measurements gave estimates of the total numbers (in tens of thousands) of fish landed in each 10-cm group. These are given in the second column of the table below.

Length (cm)	Total landed	Estimated numbers landed of each age				
		-8	9	10	11	12+
50-	91	68	23			
60-	79	21	59			
70-	168	25	114	3	25	
80-	93	6	40	17	30	
90-	45		14	11	15	5
100-	21				10	11
Total	497	120	250	31	80	16

The numbers of each age and size landed are estimated, and entered on the table; thus in the age sample 25% of the 50-60 cm group were 9 years old.

Therefore 25% of the total numbers of a 50cm group = .25 x91 = 22.75 thousand fish are 9 years old. Similarly the other entries are obtained (here rounded off) and by summing each column the estimates of the total numbers of each age are obtained. Computation is in fact easier by using a simple raising factor for each length group equal to the number landed: number in age sample, and multiplying each in the number of each age in the sample by this factor. Thus for the 50-group the raising factor (in tens of thousands) is 91/4 = 22.75, and the number of fish 8-years old and younger 3 x 22.75 = 68 (to nearest ten thousand).

If length data is being collected in any case, the advantages, in reduced change of bias, and smaller variance, of this method of estimating age composition are very great. Assuming the length sampling is unbiassed, then bias can only be introduced if, for fish of a particular size, fish of certain ages are more likely to appear in the sample. This seems unlikely - selection, either unconsciously by the scientist picking up a few fish from a pile, or consciously by the fisherman sorting out fish for different types of buyer, is nearly always related to size. It seems, however, difficult to imagine how any selection or bias could arise between two fish both say 35 cm, but one 3 years old and the other 5 years old. Much the same is true for other characteristics related to length, though possibly there might be sufficient difference in external appearance between immature and mature fish of the same length for possible bias to arise.

In example 3.3.1. the number of fish in each length-group whose age was determined was roughly proportional to the numbers landed in each length-group. This is not necessary and indeed the numbers sampled in detail at each size can be chosen according to any desired system. In the landings the younger and smaller fish usually predominate, so that in any random, or semi-random, sampling system more of these young fish will occur. Their numbers are therefore likely to be estimated with relatively greater precision than those of the older fish. However, for many purposes, e.g. estimation of mortalities, it is desirable for the abundance of as many ages as possible to be estimated with fair and roughly equal precision; this can be achieved by increasing the sampling intensity on the bigger fish.

The variance in the estimates of the numbers in each age obtained by the use of such an age-length key is given at once from the multinomial distribution. Thus if in a certain length group:

N = number of fish in the landings
n = number of fish sampled for age
np_i = number of fish in sample, age i
N_i = estimated number of fish, age i, in the landings of the given size

then $N_i = Np_i$

variance of $np_i = np_i\ (1-p_i)$

variance of $N_i = \frac{N^2}{n} p_i\ (1-p_i)$

and coefficient of variation of $N_i = \sqrt{\frac{(1-p_i)}{np_i}}$

The estimate and variance of the total numbers of a certain age landed of all sizes is given by simple addition (the variances in the different length groups can be added together because the sampling distributions in the various length groups are completely independent). The expression above for the variance assumes that N, the number of the given length group landed, is known precisely. In practice it too will have been estimated from some sampling procedure. An additional term should therefore be included, so that the proper estimate of the variance of N_i is:

$$\text{Var } N_i = N^2 \frac{1}{n} p_i (1-p_i) + p_i^2 \text{ var } N.$$

The first term is likely to be the larger, and in any case the presence of the additional term due to the variance of N does not reduce the desirability of making the first term as small as possible. It will be seen that the part of the variance due to estimation of p_i is, as usual, inversely proportional to n, the number of fish sampled, so that the precision will be improved by any increase in sampling. However, it will also be improved if p_i is large, and in fact as p_i approaches unity, the variance falls towards zero, i.e. if nearly all the fish in the length group are the same age, then the numbers of fish of that age and size are known very precisely.

In using such an age-length key essentially three choices have to be made:

(a) how to divide the entire length-composition between length-groups,

(b) How, if at all, to divide the year - i.e. should a single key be used for the whole year, or separate keys for each month or quarter,

(c) how many fish to sample in each length-group.

In choosing groupings by size of fish and season, which is another example of choice of strata of stratified sampling, it is as usual desirable that the strata are so small that there is no marked difference in distribution (here in the proportion of the various ages) between possible sub-divisions of any single strata. There is also the desirability of decreasing the variance by making p_i as large as possible, i.e. of making as far as possible each length group consist of a single age. This may well be possible at least for the smaller sizes where the fish grow fast and fairly uniformly. For instance the following table gives quarterly age-length keys for the smaller sizes of cod caught in the North Sea in 1960, in terms of the numbers of fish of each age in the samples.

Length Group	30-39				40-49				50-59				60-69				70-79			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1 year old			12	56			1	23			1									
2 years old	36	26	36	8	62	51	73	50	15	41	54	63		2	13	43				
3 years old	1	4			4	1	3	2	18	9	6	2	19	21	25	17	4	28	27	30
4 years old									2				1	3		11	20	7	7	6
5 years old													1	1			9			
Total	37	30	48	64	66	52	77	75	35	50	61	65	21	27	38	61	33	35	34	36

So far as choice of seasonal groupings is concerned, the two smaller sizes in particular show very big changes during the year, so that a single grouping for the year is unsuitable. However, there is not much difference between successive quarters (except for the smallest size), so that there is probably even less difference between months. Quarters, therefore, are at least a fine enough subdivision of the year, and even two half-yearly age-length keys would be reasonably satisfactory. For the big fish, 70-79 cm, and still more for the bigger fish for which data are not given in the table, there is little seasonal change. For these, therefore, a single annual age-length key would be suitable. There is no basic reason why different periods for different length-groups should not be used in this way. However, in practice it may be more convenient to use the same time intervals for all length groups - here then two half-yearly age-length keys seem best.

Between length-groups there are no very abrupt changes. Thus the proportions of the 3 year old fish in the first quarter, for each of the five length-groups, are 0.03, 0.06, 0.51, 0.90 and 0.12 respectively. This suggests that there would be no very big differences between the age-distribution of fish say in the 5 cm groups 40-44 and 45-49, and this is in fact confirmed by analysis of the original data. Thus 10 cm length groups are sufficiently small. However, there are sufficient differences between 10 cm groups to show that any coarser grouping, e.g. in 15 cm or 20 cm groups, would not give such precise estimates.

The optimum numbers sampled in each length group are likely to depart quite widely from the proportions likely to be obtained by random sampling from the landings as a whole. In the landings the majority of the fish are likely to be concentrated in the smaller length-groups. The number of fish of the ages represented in these groups will therefore be known very precisely, while the estimates of the numbers of older fish will have considerable variance. For many purposes it is probably desirable to have the numbers of at least the more important ages estimated with roughly equal precision. For instance, if the mortality is being estimated as the ratio of 5 year old to 4 year old fish, it is little help if the number of 4 year old is known very precisely if at the same time the number of 5 year old fish is only badly estimated. A reasonable system of sampling would therefore be to take equal numbers for age analysis from each length-group.

In fact, even then the younger ages may be estimated more precisely, as at least for fast and uniformly growing fish each of the smaller length-groups may be comprised of almost entirely one age-group, thus reducing the term $(1 - p_i)$ in the expression for the variance, $\text{var } N = N^2 \frac{1}{n} p_i (1 - p_i)$. In contrast the larger sizes will contain fish of several different ages.

Example 3.3.2.

Using the age-length data for cod given in the previous table, and the fact that the estimated number of fish in thousands landed each quarter was as follows:

Quarter	1	2	3	4
30-39	202	275	160	270
40-49	327	675	488	508
50-59	107	200	394	673
60-69	109	116	205	329
70-79	43	70	139	104

estimate the number of fish of each age, 1 to 5, less than 80cm, landed each quarter, and the total for the year. Also estimate the total for the year from a single age-length key for the year.

Assuming the numbers landed of each size are known precisely, calculate the variances of the estimates of the numbers of each age, using quarterly and annual age-length keys.

Data from several sources

Apart from their use for improving the efficiency of sampling in a single simple fishery, age-length keys (or maturity-length keys, etc.) are invaluable in any mixed fishery, where the same stock is being exploited by a number of different gears, or different countries. However similar two groups of vessels may appear to be, it is very unlikely that even when fishing the same stock their catches or landings will have precisely the same composition, in terms of length, age, etc. Differences may arise through the more obvious types of selection, e.g. escape of the smaller fish through the meshes of a trawl, or other types of gear selection - or, and possibly most often, through differences in the precise grounds fished. Whatever the causes, the result is that samples taken from catches of one type of gear, or by ships from one port, or one country, cannot be taken as representing satisfactorily the landings by other gears, or in other ports or countries. The differences between the landings are essentially in the sizes of fish, and the relation between say age and length is the same for all - that is, just as any selection by a scientist sampling the fish after landing is related to length, and not to the age of the fish of a given length, so will the gear select by size of fish, and will not in general be able to descriminate between say 2 and 3 year old fish of 30 cm. There are, of course, exceptions; gear selection in the strict sense is unrelated to age, for a given length, but two ships can fish grounds with different size composition of fish, and with different growth rates.

It should be noted that if two grounds have noticeably different growth rates, then the mixing between them cannot be very rapid, and they may have to be treated separately, not only in the sampling, but also in subsequent analysis.

In general though it can be taken as a rule that samples of length composition should be taken from all groups of vessels exploiting a stock of fish (from each gear, each country, etc,), age-length keys can be derived from only a section of the fishery, and then be applied to the length-compositions of all landings.

Example 3.3.3.

During 1960 seiners landed at Grimsby from the North Sea the following numbers of cod (in thousands) under 80 cm

Quarter	1st	2nd	3rd	4th
30-39	2	2	5	18
40-49	12	55	138	87
50-59	6	66	196	70
60-69	5	43	80	14
70-79	3	42	91	6

Using the age-length keys derived from the trawler landings, estimate the numbers of fish at each age.

Ideally, of course, age data, or samples for constructing an age-length key, should be taken from all, or as many sections of the fishery as possible. Even then it is best to combine all age-length data together to give a single age-length key, which can then be applied either to the total landings of each length-group, to give the total age composition, or to each section (e.g. landings by a particular gear) separately to give the separate age-composition. This of course is essentially a problem in stratified sampling; each length group can be considered as an independent population, for which the number at each age has to be estimated. Stratification, i.e. treating each section of the landings separately, will not be an advantage because the distributions of age (for a given length) within each section is the same, and will give increased variance unless sampling is done precisely in proportion to the numbers in each section - which is unlikely.

Such pooling of data of age at length is likely to be particularly valuable in an international fishery. Age determination, or similar work, is not only time-consuming but also requires a fair degree of skill and experience. Not all countries exploiting a stock of fish may be able to collect age data -e.g. due to shortage of suitable staff, but all should be able to sample for length composition. Age-data for the entire landings may then be obtained, either by the appropriate international agency using length data from all sources, and all available age-length data, or nationally, provided that where age data are available they are published in the form of age-length keys.

4. SAMPLING THE POPULATION IN THE SEA

4.1 General problems

The previous section is concerned with the sampling of a clearly defined and observable group - the fish caught or landed by the fishing fleet. The sampling problem is relatively straightforward, and theoretically it is possible to achieve complete accuracy by sampling the complete catch - and indeed this can sometimes actually be done, for instance for whales, where the length of each one caught is recorded. The research worker is however particularly interested in the natural population from which the catch comes - though it must not be overlooked that even when the catch is far from representative of the natural population information on the age and length composition , etc. of the catch is of great importance as these are the fish that are actually being removed from the stock. The estimation of what is in fact the population of fish or other organisms in the sea is of course a major research task, and much of it is outside the scope of this manual, but there are many possible applications of straightforward statistical and sampling techniques. Many of these applications are relevant not only to the estimation of stocks of commercial fish but also to other fishery and oceanographic problems, e.g. plankton surveys. The first stage in the sampling programme, either in planning the work of a special research vessel, or the analysis of catches of the commercial fishermen concerns the type of gear - and for many, if not nearly all, fishery workers, the most important single class of gear used in sampling the natural population is the fishing gear of the ordinary fisherman.

All, or nearly all marine sampling instruments are highly biassed, and sample representatively only a small proportion of the true population. For instance, a fine meshed plankton net will allow the larger and more active planktonic animals progressively to escape, while a coarser meshed net will seriously underestimate the quantities of the small larval plankton. Again, with a trawl, the small fish can escape through the meshes, while some of the most active swimmers may escape from the path of the trawl completely. The actual gear used has therefore to be carefully chosen in relation to the particular organism being studied, with the corollary that estimates for other organisms will almost certainly be biassed.

A distinction may be drawn between gears which can estimate numbers on an absolute scale, and those which can give only relative numbers, or indices of abundance. Thus catches of fish eggs with a Hensen net can, with suitable precautions, be expressed as numbers below a square metre of surface, or numbers per cubic metre. Catches with a trawl cannot usually be expressed more precisely than numbers per haul of standard duration; if this catch per unit effort is twice that at some other time, or in some other place, then it may be presumed that the fish are twice as abundant, but the absolute scale of numbers, or numbers per unit area, is not known. The sampling power of the gear - trawl, seine, etc. - that is the amount taken on a given density of fish, is not likely to be the same for each species of fish; apart from the more obvious forms

of selection, determined to a large extent from the geometry of the gear - for example, escape of small fish through the meshes of a trawl or seine, or selection by a gill-net, both of which are determined by the relative circumferences of fish and mesh - the type of fish caught is to a large extent governed by details of the exact rigging of the gear, and behaviour of the fish.

Selection by size of fish but not selection due to differences in behaviour may be reduced by using a range of mesh sizes. For instance a fleet of gill-nets of suitably graded mesh sizes might be used in two locations in an African lake. In the first 100 predatory Hydrocyon and 50 herbivorous Tilapia were caught; at the second, in the same fishing time, 50 Hydrocyon and 150 Tilapia were caught. These data, with due reservation concerning random sampling variations, show that the density of Hydrocyon in the first area is twice that in the second, but that the density of Tilapia is three times as high in the second area as in the first. However, because the active, predatory Hydrocyon is much more likely to encounter a gill-net it is not possible, without first having some information from which the greater vulnerability can be expressed in quantitative terms, to compare the densities of the two different species.

Similar sorts of selection - that difference in catches on a given density of fish - can occur with a single species of fish at different times. Thus there may be definite seasonal changes in catches not apparently connected with real changes in total stock density, or, on a smaller time scale, differences between catches during day and night. Such differences should not introduce errors into the sampling system if it is properly designed. Thus if much bigger catches are taken by day than by night, then the proportion of fishing by day should not be different in different areas or in different years - otherwise an area might appear to have the highest density of fish merely because relatively more fishing by day was done in that area. The actual proportion does not matter - it may be both comfortable and convenient for a research vessel to do all its fishing in daytime (proportion of day fishing always = 1) and none at night, while fishing by commercial vessels may be continuous in all areas (proportion of day fishing = 0.5). Seasonal changes may be eliminated in the same way. Again the seasonal distribution of fishing should be the same in different years, or in different areas; for research sampling this may be rather more difficult because of other possible work to be done, while the seasonal pattern of commercial fishing may change with changes in the abundance of fish, market conditions, etc. For instance, the decline in a stock of fish previously fished throughout the year may mean that it is only profitable to fish it at the season when catches are highest. If this change in the season of fishing were not detected and taken into account, the decline in the stocks, as measured by the catch of commercial fishermen, would be markedly underestimated.

4.2 Surveys with a research vessel

Once the most suitable gear has been chosen, the chief difficulty in sampling the natural population lies in the generally very patchy distribution. Very big differences in samples occur with virtually all the organisms a fishery worker is likely to wish to sample (fish, plankton, benthos, etc.). These

differences are both marked and consistent ones between large areas, or between seasons, and apparently random variations at one time and position. Thus catches of plaice are, on the average, much higher in the southern and shallower part of the North Sea than in the north, and within each region there are well known grounds giving good fishing. However, repeated trawl hauls on the same ground under apparently the same conditions may give catches differing by a factor of 2 or more. This rather large variance and also the usually high cost of a single observation mean that it is particularly important to ensure that the sampling system used is as efficient as possible - i.e. gives the smallest variance for a given cost. As usual this is likely to be achieved firstly by using some suitable stratification - dividing the area in sub-areas within which conditions are more nearly uniform - and secondly by taking as many samples as possible, if necessary at the cost of reducing the size of sample.

The first step in stratification will be into broad geographical regions: within these more detailed sub-division is possible. For bottom living animals (including demersal fish) depth is a very important factor, and stratification by depth zone is very likely to give improved accuracy. Other stratifications may be determined before starting the survey from other elements in the environment - current systems, or during the survey on the basis of the catches being made. For instance, much of the efficiency possible in stratified sampling will only be achieved if the sampling effort is properly divided between strata, i.e. if most samples are taken where the variance is greatest - normally where the abundance is greatest. Thus the early samples taken duing a survey may be used to determine a rough pattern of abundance and distribution of some organism, from which the areas can be determined in which more intensive sampling is desirable. In its simplest form, this merely involves stopping a line of stations as soon as one, or perhaps two, blank hauls have been made, and otherwise maintaining the same pattern of lines of stations within the survey area. The efficiency will be much further increased by varying the intensity of sampling - for instance, having lines of stations perhaps 40 miles apart at the fringes of the distribution, 20 miles apart nearer the centre, and 10 miles or less where the abundance is highest.

The similarities between surveys by a research vessel and any other type of sampling scheme are often concealed by the method of analysis using contour lines at certain levels of abundance. Drawing such contours has very definite value as a descriptive procedure for the pattern of distribution of any organism, and one which may be compared with other distributions - of hydrographic factors, food, etc. The contour charts may also be used to estimate the total abundance, by calculating the area between each pair of contour lines, and multiplying this by the value midway between the contour lines.

This procedure is not far different from normal stratified sampling, but to use it to obtain a precise estimate requires that the contour lines be drawn with some care and the areas calculated quite precisely, both of which can be time consuming, and which tend to attach a greater degree of accuracy to the precise position of a contour line than is normally justified. An alternative procedure, which is more clearly the same as methods of stratified sampling

described in earlier sections, is to divide the region into subdivisions of known area (calculation of the area may be made less laborious if at least the majority of the dividing lines run north-south or east-west), estimate the density in each subdivision as the mean of the observations (e.g. catch per haul) made in that subdivision, calculate the abundance in each division as the product of area x density, and hence estimate the total abundance as the sum of the abundances in each sub-area. A special case of this procedure occurs when the survey has consisted of a uniform grid of stations, say 10 miles apart. Then each station can be considered as a sample from the ten-mile square of which it is a centre, and the estimate of the mean density is the mean of the densities at the individual stations. For any one organism, unless very uniformly distributed, this pattern of sampling is inefficient, possibly greatly so, because too big a proportion of the sampling effort is made in areas where the density is low. It is most likely to be useful when making a whole range of observations (hydrographic, phytoplankton, zooplankton), each with a different distribution. The difference in the estimates obtained from contouring and from the direct means of observations (stratified as necessary) is probably not much, as can be checked by using both methods on the same set of data. As the second method makes use of the full information available, and is less likely to contain computational mistakes, it is probably rather more precise than the contouring method. In such survey work it is therefore likely to be best to use contours to give a general description of the situation, and the straightforward stratified sampling to give the quantitative estimates.

As usual avoidance of bias is a major sampling problem. Bias introduced by the type of gear used has already been discussed, and is essentially a problem of design of fishing gear, etc., and outside the scope of this manual. Bias can also be introduced through the choice of sampling position. Strictly, of course, unbiassed sampling is best achieved by truly random sampling. The practical difficulties in using a set of sampling stations each chosen randomly and independently are very great, and the time spent moving from station to station would be greatly increased. A systematic grid, with lines of stations at regular intervals, is the standard practice, and only the precise position of the origin of the grid need be taken at random. Further, because the lines of latitude and longitude are, so far as any small area is concerned, artificial concepts, they may be considered as random relative to the natural population, and thus form a convenient basic grid for the lines of samples. Some care should be taken when there are definite trends running east and west (or north and south). Here, and anywhere where there is a marked trend, detailed stratification is particularly useful. Suppose there is a coastline running north and south and say the numbers of fish caught on hook and line decrease with increasing distance from the coast. Then the whole area should be divided into strips parallel to the coast, and an appropriate sampling pattern would be lines running east and west, perpendicular to the coast, with each line having one or more stations in each strip.

The sampling pattern particularly to be avoided is one in which the sampling positions, within the finest stratification used, have been chosen with reference to the characteristics being sampled. For instance, it is tempting, when fishing to determine the relative abundance and other characteristics of a fish stock, to

do so at points where it is known that good catches, and therefore large samples (e.g. for racial analysis), are probable. So long as the habits and distribution of the fish are unaltered, the data will give reliable estimates of the relative abundance of the fish. However, if the distribution of the fish changes, without altering their total abundance, the catches will decrease, apparently as a result of a decrease in the stock abundance. This does not mean that, for example, experimental trawling should not be made on good trawling ground, in the sense of grounds where the bottom is smooth and sampling with a trawl can be done without undue risk of damage to the gear; but a ground should not be chosen merely because it is a good ground in the sense of having plenty of fish. It must be emphasized that this danger only refers to the choice of sampling position within the finest stratification. Overall sampling should be concentrated where the abundance is greatest; that is the stratifications with the greatest abundance will have the most intensive sampling, though within each stratification the distribution of sampling will be random, or nearly random - e, g, on a grid.

The intensity of sampling - how far apart to have the stations - is mainly determined by three factors: the variance between estimates from different stations, the time taken to move between stations, and the time spent sampling on any one station. The problem is therefore analogous to the two-stage sampling discussed previously, and as before, the exact solution can only be determined from data of the particular problem concerned. Increased variance between samples requires more samples, increased time on one sample implies fewer samples. The similarity with previous two-state sampling problems is even closer when there is a possibility of varying the size of sample at any one station, e.g. by taking more than one haul with a plankton net, or using a two- or three-hour hauls with a trawl, rather than a short, perhaps half-hour tow. Factors which have then to be considered are the variance between different samples at the same station (for long trawl hauls the difference between the catch in a one-hour haul and the extra fish caught in a two-hour haul), and the time spent at a station, but not actually sampling - e.g. shooting and hauling the trawl. If the within-station variance of the non-sampling time on station is small, then it is best to take a small sample (a single plankton net haul, or a short trawl haul). Conversely, if successive hauls at the same station differ almost as much as hauls five or ten miles apart, or if it takes a long time to stop the ship and prepare the gear for sampling, or to shoot and haul the trawl, then a large sample (several net hauls, or a two-hour trawl haul) is likely to be best. On the whole small samples are likely to be best to use unless there is information to suggest otherwise - that is where the sampling gear takes discrete samples (vertically hauled plankton nets, bottom samplers, etc.) it is usually best to take one sample at each station. For ordinary trawls the best is probably a duration of perhaps half an hour, longer in deep water when hauling takes longer, and for other gears (e.g. Agassiz trawls, towed plankton nets) a short haul at each station.

The natural pattern of sampling is on a grid of stations, spaced at regular intervals along lines at regular intervals. There is no over-riding reason why these intervals should be the same - the stations could be 5 miles apart, and the lines 10 miles apart, and this difference may be used to improve the sampling efficiency. Much of the time spent by a research vessel during a survey is

spent steaming between stations. This time is not increased greatly by adding more stations in a line, but is at once increased if an extra line of stations is added. The greatest number of samples in a given time at sea is therefore obtained by having stations close together along lines which are far apart. In any very extreme form this sampling pattern would generally be inefficient in terms of the information obtained from a given number of stations, because there would not be much difference between successive stations in the same line, but big differences between lines, with the gap between the lines being poorly sampled. A not very infrequent exception is the situation where the changes in one direction (say north-south) are greater than those in the direction at right angles (east-west). If the lines run in the direction of greatest change then the differences between successive stations close together in the same line may be the same as differences between adjacent lines which are some little distance apart.

So far only a single survey has been considered. Usually a research worker is concerned with changes in the abundance and distribution of an organism in time as well as in space. The question therefore arises whether to use the available resources, not only of ship's time, but also for subsequent analysis of the material collected, to obtain a small number of detailed surveys, say at monthly intervals, or rather more, but less detailed surveys, say at weekly intervals. Statistical considerations alone are not sufficient to answer this question, but they would suggest that the larger numbers of surveys are better, provided that each individual survey is not too scrappy.

4.3. Commercial catches as samples of the population

Information on commercial catches is of great importance in itself in any study of exploited fish stock, because this is what is being taken from the stock. It is also most important in being one of the best sources of data on the fish population itself. In a fishery of any appreciable magnitude the number of operations made by commercial fishermen, (e.g. number of nets used), and the numbers of fish caught, will be vastly greater than can be achieved by a research vessel. If there is a reasonable system of collecting statistical data on commercial catches and fishing effort, each of these numerous operations will provide a sample of the exploited population. With such a large number of samples the resultant estimates will have small variance. Unfortunately the chance of bias is correspondingly large in estimates of both abundance and composition of the stock. This bias can, however, be in many situations reduced or even eliminated.

4.3.1. Indices of abundance

A fisherman will always fish where he thinks fish will be abundant. His catch per unit effort will therefore be higher than that obtained by, say, a research vessel fishing randomly. This does not affect the value of his catch per unit effort as an index of the abundance of the fish stock - it merely alters the value of the coefficient of proportionality relating catch per unit effort to stock abundance. Mathematically, denoting this coefficient by q, we can say that q (research vessel) is less than q (commercial fisherman), but both can be equally valid. Bias arises

when q varies. Such variations can be divided into:-

(a) Short-term fluctuations in time, of more or less regular periods - day and night, or seasonal changes.
(b) Long-term trends.
(c) Irregular changes in time.
(d) Changes correlated with fish abundance.
(e) Changes correlated with the amount of fishing.

The causes of these changes can themselves be divided into fairly distinct groups:-

(i) Changes in the vessel and gear - that is, in the fishing power of the fishing unit.

(ii) Changes in the distribution of fish and fishing.

For example, improvements in the gear will produce a general increase in the catches on a given density of fish - i.e. a long-term trend in q; so also will improved navigational or fish-finding devices which enable the fishermen to concentrate better on the high densities of fish.

The first classification is the most relevant so far as the ultimate use of catch per unit data is concerned, If, for instance, the effect of changes in the amount of fishing on the stock is being studied, then any undetected type (e) or possibly type (d) variations could nullify the whole work. Similarly any historic survey of a long series of past records must take particular account of any variations of type (b). The second classification is the more relevant to the study of the variations themselves and to their detection and possible measurement. The study of changes in vessel and gear is, in the main, not a sampling problem, but a question of the proper measurement of fishing power, and fishing effort, and therefore outside the scope of this manual. More directly concerned with sampling techniques is the problem of the distribution of fishing power and fishing effort. This can be divided into distributions in time and space.

The shortest time scale over which changes are important is the day. For instance, trawl catches during daylight and darkness may differ quite considerably, but still the catch per hour during daylight will be proportional to the fish abundance, as also will the catch per hour during darkness, though with a different constant of proportionality. The catch per hour based on a small number of trawl hauls would therefore not provide a good index of abundance unless it was also specified what proportion of hauls were made in daylight and in darkness. Commercial trawling will often be carried out more or less continuously, so that the proportion of the total amount of fishing made in the darkness and in daylight will be the same from year to year. Diurnal variation is therefore only important in the analysis of commercial catches if there is any change in the daily habits of the fishermen - for instance, if night catches are normally poor they may cease to fish at all during the night at times when the general abundance of fish is very low.

Similar considerations may be applied to other short term variations - for example, short term differences in catches per unit effort due to differences in weather may be assumed to become small when considering the mean catch per unit effort over, say, a year (though there may well be appreciable differences in the average weather conditions between short seasons of, say, a few weeks).

Seasonal fluctuations might also be neglected on the same grounds, but there is more scope for fishermen to change their seasonal fishing pattern than their practice over a day - thus when fish are not abundant fishing for a given species may be carried out only at the peak of the season, when catch per unit effort is highest, but when fish are abundant fishing for this species might go on for most of the year. The nett effect, of course, is to reduce the variation in the apparent catch per unit effort. This possible error can be reduced by suitable stratification. Instead of using as an index of abundance the annual catch per unit effort calculated as the total year's catch divided by the total year's effort, the year can be divided into periods, say months, each of which is, as usual, sampled separately. Then, there will be twelve independent indices of abundance, each calculated as the month's catch divided by the month's effort. Any one of these could be used as an index of abundance, or a single mean calculated to give the index for the year. This might be the simple, unweighted mean, or more weight could be given to the main fishing seasons. These weights must, of course, be the same for each year, so that it would be wrong to choose the weights as proportional to, say, the amount of fishing which would vary from year to year, but they could be chosen as proportional to the average amount of fishing in each month over some fixed period of years.

Ex.4.3.1. Monthly catches of Bagrus spp. from Lake Menzala, Egypt, during the two years 1956-7 and 1957-8, and the corresponding fishing efforts were as follows:-

Month	June	July	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May
1956-7 Catch (tons)	21	6	15	132	210	57	34	5	9	8	14	15
Effort	12	14	20	21	22	14	11	11	8	7	7	6
1957-8 Catch (tons)	9	3	15	210	288	20	32	4	3	2	3	10
Effort	6	7	25	42	36	10	8	7	3	2	2	5

Calculate the catch per unit effort in each month; calculate an index of abundance for each year (a) as the simple arithmetic mean of the monthly catch per unit efforts. (b) giving weights to each month of 6, 7, 15, 20, 19, 8, 6, 6, 4, 3, 3, 3, respectively (roughly proportional to the average fishing). Compare these with the ratio of total year's catch to total effort. Compare the abundance of Bagrus in the two years (a) from any of the indices above, based on stratified sampling. (b) as catch per unit effort in each year taken as a whole.

There is no reason for the time periods to be equal. In the example above, strata each of one month were used, as these were the basic periods used in collecting the data, but it might be better to group some months together in the seasons of poor fishing, so as to have more or less equal amounts of fishing in each division. As usual the most important criterion in grouping is that within any one division the quantity being sampled (in this case catch per unit effort) is as uniform as possible. Thus it would certainly be correct to group data for February and March, and for April and May together while other groupings, e.g. July and August, are also possible. Conversely it may be better to divide the months of September or October into two fortnightly periods.

The example also demonstrates that the catch per unit effort in a single month can provide a reasonable index for the abundance over the year - that is, the changes in the catch per unit effort in, say, August of different years are proportional to the changes in the stock. Therefore, while it will in general be obviously better to base an index of abundance on catches over the whole year, it is perfectly possible to use data for only part of the year, either because little fishing is done at other times, or because in the rest of the year the catch per unit effort is not likely to be reliable (the fishermen are perhaps fishing mainly for some other species).

The technique of stratified sampling is also useful in producing indices of abundance undistorted by changes in the distribution of fishing in space. The whole area being fished is divided into sub-areas, and instead of taking as an index of abundance the ratio of total catch to total effort, the mean (weighted, if necessary) of the catches per unit effort in each sub-area is used. Thus the main trawling grounds on the west coast of Greenland are a series of banks - Banana Bank, Danas Bank, Fyllas Bank, etc. A reasonable index of abundance for the west Greenland stock of cod would therefore be the mean of the catch per hours trawling on each bank. As the banks are of different sizes, a better index might be obtained by weighting the catches per unit effort by the area of the bank concerned (for catch per unit effort measures density of fish, so multiplying by area should give abundance). The method clearly cannot be used directly unless data are available for each bank - i.e. in the usual terms a sample must be taken from every strata. This sets a very definite lower limit to the size of area which can be used. In fact, it means that stratification by area can be used to study changes in what might be called fishing strategy, but not in fishing tactics. Good strategy might be termed the success of the fishermen in moving to the particular Greenland bank where fishing is at that moment best - perhaps as a result of wireless information from other ships, while tactics is the ability to fish at the best spot on the bank once he has arrived there - e.g. by direct detection of the fish by echo-sounding, or by more precise navigation to keep exactly on the best spot for fishing - e.g. again by echo-sounding, this time purely for determining depth, or by radar, Decca navigator, etc. Such improvements in fishing tactics can be among the main contributions to a long-term increase in q, the ratio of catch per unit effort to fish density, and probably the one that is most difficult to detect and measure in quantitative terms. Further discussion of this problem is one that is outside the scope of a manual on sampling.

Stratified sampling by area to eliminate bias due to changes in fishing strategy is particularly useful where a fishing fleet catches two or more species, whose distributions differ but overlap. Changes in the relative abundance or market demand for the different species may cause quite different patterns of distribution of the fishing effort.

Example 4.3.2.

The table below is an extreme simplification of English North Sea trawl data and represents the effort and catches of plaice and haddock in two years; plaice being predominantly a southerly fish, and haddock found in the north. The area has been divided into 16 sub-areas in each of which the data of catch and effort recorded separately.

	1960				1961			
Effort	5	6	6	3	16	17	13	14
Haddock	50	48	60	24	208	238	195	168
Plaice	0	12	6	0	0	17	13	0
Effort	8	7	9	8	13	12	13	10
Haddock	40	49	54	48	130	132	91	80
Plaice	16	0	27	8	13	12	26	0
Effort	10	13	11	14	9	9	(8)	6
Haddock	40	65	33	56	45	63	(32)	48
Plaice	40	39	22	42	18	18	(8)	18
Effort	14	15	16	15	5	5	6	4
Haddock	28	0	16	15	10	5	6	4
Plaice	84	90	48	45	25	15	12	8

(i) Calculate for each year total catch of each species, and the total effort, and hence the ratio total catch/total effort.

(ii) Draw up a chart for each year showing the catch per unit effort of each species in each sub-area.

(iii) Hence calculate an overall density index for each species, i.e. the mean catch per unit effort. Compare the change in density between the two years as measured by the ratio total catch/total effort, and by the mean catch per unit effort.

Another type of stratification is sometimes possible in a fishery for more than one species, when there is sufficiently little mixing between the species for catches at any one time - e.g. from a single trawl haul - are predominantly of one species, - even though both are present in the general area. Landings by individual catch can therefore be divided according to the species primary fished for.

Example 4.3.3.

German trawlers fish at West Greenland for both cod and redfish. Their catches in tons and fishing efforts during 1958 and 1959 were as follows (data taken from I.C.N.A.F. Statistical Bulletins).

	Primary species	Days fished	Catch of cod	Catch of redfish
1958	Cod	1,337	26,247	1,754
	Redfish	385	1,277	9,457
	Mixed	199	2,386	1,969
	Total	1,921	29,910	13,170
1959	Cod	645	12,336	1,087
	Redfish	690	2,705	15,683
	Mixed	169	2,372	2,062
	Total	1,504	17,413	18,832

Estimate, from the catch per unit effort of cod by the trawlers fishing for cod, and of redfish by trawlers fishing for redfish, the changes (as percentages) in the densities of the stocks of cod and redfish between 1958 and 1959. Compare these with the changes in the catches per unit effort of cod and redfish by all vessels taken together, and also with the changes in the catches per unit effort of cod by the redfish vessels, and of redfish by the cod vessels.

4.4. Composition of the stock

Commercial landings will always to some extent be biassed samples of the complete stock - fish that are too small, or in too poor condition to be sold, as well as unwanted species, will be rejected from the catches. The catches themselves will be biassed, not only through the obvious geometrical characteristics of the gear - e,g, escape of small fish through the meshes of a net - but also because the fishermen will prefer to fish in areas where fish of the most valuable or abundant size, etc. are present. Thus British trawlers fishing in the North Sea have a mesh size of around 80 mm (i.e. a little above the legal minimum of 75 mm);

this mesh will retain most plaice greater than 20cm in length and virtually all greater than 22 cm. The minimum legal landings size is 25 cm, so that we can hope to use the commercial trawler landings to give information on the sizes of plaice above, say 28 cm (as rejection by the fishermen of undersized fish will not be precisely on the minimum size limit). Thus, for a particular ground the population (say numbers per square mile), and the catch and landings by a trawler from ten hours' fishing of each size of fish, might be as follows.

Length (cm)	5-10	10-14	15-19	20-24	25-27	28-29	30-34	35-39	40-44	45+
Numbers per square mile (A)	500	700	820	780	360	230	180	80	40	20
Numbers caught	-	10	120	360	180	115	90	40	20	10
Numbers landed (B)	-	-	-	10	150	115	90	40	20	10
B/A	0	0	0	0.01	0.42	0.5	0.5	0.5	0.5	0.5

The bottom row shows that the ratio of numbers landed to numbers per unit area is constant above 28 cm, so that the landings give an unbiassed picture of the length composition of the stock above this size. The landings per unit effort are equal to the average numbers within a certain area - half a square mile in the example above. The size of this area will not be known, which is why the catch data are used to give indices of abundance, not abundance in absolute terms, though its order of magnitude can be gauged from the geometry of the gear used. Thus a trawl might catch most of the fish in an area equal to the width of the net (or the distance between otterboards) times the distance travelled. However, on the one hand many fish may escape over or around the net, and on the other the trawl may be fishing on a density of fish higher than the average.

The landings in the example above are only representative of the stock on the grounds being fished. The sizes of plaice, like most other fish, vary quite markedly from ground to ground, the smaller fish being most abundant in the shallower grounds along the coast, and the bigger fish in the deep water. The size composition of the fish on different grounds, again in terms of numbers per square mile, might therefore be:-

Length (cm)	-10	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45+
Shallow	500	700	820	780	590	180	80	40	20
Moderate	-	50	150	320	400	200	100	70	40
Deep	-	-	20	50	150	180	120	80	60

Obviously the landings in the example above (from fishing on the shallow grounds) give a biassed picture of the length distribution of the fish above 28 cm in the stock as a whole, for example greatly underestimating the importance of the fish over 45 cm (a ninth of those between 30 and 34 cm, compared with a third in deep water). This bias will still occur, even if fishing is done in all zones, unless the amount of fishing is exactly in proportion to the area of each zone.

Example 4.4.1.

Assuming the areas of the zones above are: shallow, 4,000 sq. miles, moderate 8,000 sq. miles, and deep 5,000 sq. miles, calculate the total numbers of each size group in the stock. If in ten hours' fishing a trawler catches a quantity of fish equal to half the average numbers per square mile, calculate the total numbers landed in each length group above 30 cm, if 100,000 hours' fishing was carried out, distributed as follows:-

(a) Shallow 50,000 hours, moderate 45,000 hours deep 5,000 hours.

(b) Shallow 10,000 hours, moderate 40,000 hours deep 50,000 hours.

Compare the length distribution (as percentages) of fish over 30 cm, of the stock and landings.

Unbiassed estimates, or at least estimates with the bias reduced as far as possible, can be obtained by suitable stratification according to the area fished. The usual principles of stratified sampling apply, except that there is not full control on the source of the samples. The fishermen will fish when they want to, and not where their catches will give the best information for the scientist, and even the sampling by the scientist of the landings may not be fully controlled. For instance, trawlers may arrive at Grimsby during the night, when their crews will go home immediately. The fish are unloaded and set out for the auction very early in the morning, and have to be sampled then, and it is not possible to interview the crew, and find out where the ship has been fishing until later, after the sampling has been done. Thus it will often not be possible to apportion the sampling effort (either by the commercial fisheries sampling the stocks, or by sampling the commercial catches) between areas (stratification) in precisely the most efficient manner. The departure from full efficiency may not be large because most catches, and therefore most samples, will tend to come from the areas where most fish are. Because there are probably less differences between different places in the sizes of fish than in the density of fish, and because fewer samples of size distribution are available, the divisions of the whole area for stratification will probably be rather larger than those used to estimate an index of the total abundance. Usually the regions used in studying the length composition will be formed by groups of two or more of the areas used in the analysis of catch and effort statistics, the choice of grouping being as usual to group together areas with similar length compositions, and to ensure that there are some samples from each strata.

These stratifications by area of catch, though essential for estimating the composition of the stock, may also be conveniently used for estimating the catch. Thus the procedure suggested earlier, for landings with no sorting into size categories was:

(1) Measure a sample from the landings of one ship.

(2) Raise the sample to the total landings of the ship (i.e. multiply the numbers in each size-group in the sample by a raising factor equal to the weight of the total ship's landing divided by the weight sampled.)

(3) Add together landings by all sampled ships.

(4) Raise to total landings for the period.

The procedure would now be:

(1) Measure a sample.

(2) Raise the sample to the total landings of the ship, (as before).

Then, for each stratification separately:

(3) Add together landings from all sampled ships fishing in the particular sub-division.

(4) Raise to total landings from the sub-division.

Then the total landings for the whole region are obtained by addition of the totals for the individual sub-division from (4). To estimate the composition of the population, either procede as follows, if the efforts corresponding to the landings in (4) from each sub-division are known:

(5) Divide landings from the sub-division obtained in (4) by the effort in the sub-division, to give the catch per unit effort of each length-group.

(6) Multiply by the area of the sub-division (to convert from units of density to units of abundance).

(7) Add together all sub-divisions. This gives numbers proportional to the overall abundance of each length group, but if desired.

(8) Divide by the total area, of all sub-divisions. This produces numbers which are again in the same scale of catch per unit effort (e.g. catch per 100 hours' trawling) as (5) - in fact, they are the weighted means of the numbers in (5), the weighting factors being the areas of the different sub-divisions.

Alternatively, if the effort corresponding to the total landings is not known, so that the catch per unit effort of each length group cannot be calculated directly, then the percentage length composition of the landings from each sub-division can be calculated. The percentage of any length group in the stock can then be estimated as the weighted mean of the percentage of this length group in the landings from each sub-division. The weighting factors must be the numbers of fish (or strictly the numbers of fish of a size that may be caught and landed) in each sub-division - i.e. the density in numbers, times the area of the sub-division. The density in each sub-division may be estimated from such data on catches per unit effort as are available.

4.4.1. Fishing by more than one fleet

If more than one set of vessels is fishing the same stock of fish (different types of gear, or vessels from different countries), then it may be possible to obtain separate estimates of the composition of the stock, which should be combined in some way to give a simple 'best' estimate. Provided that the differences are not too marked, and are no more than might be expected from the sampling errors, then this can be treated as a simple statistical problem. The combination should be done within the regional stratification discussed in the previous paragraph - i.e. a single estimate of the composition of the stock within each sub-division is obtained from all available data, and these are then combined to give the estimate for the complete stock. Within each sub-division the estimates from the various sets of vessels should be combined by taking the weighted mean, using as weighting factors numbers inversely proportional to the variances of the different estimates. These variances will not usually be known very precisely, but will be nearly inversely proportional to the number of samples taken, or number of fish measured from the landings of each group of vessels - i.e. the weighting factors may be taken as proportional to the number of samples or number of fish measured. If it is possible to use a standard unit of effort, and of catch per unit effort, for all groups of vessels, then it is best to express the estimated composition of the landings of each, and hence the combined estimate of the composition of the stocks, in terms of catch per unit effort. Suppose, for example, two groups of boats are fishing for anchovy in a certain area, both using the same type of seine net, so that the unit of catch per hour is the same for both groups. Ten samples were taken from the first group, from which it was estimated that 160 fish of 11 cm were caught per hours fishing; from eight samples from the second group it was estimated that 250 fish of 11 cm were caught per hours fishing; the combined estimate is therefore:

$$\frac{1}{18} \times (10 \times 160 + 8 \times 250) = 200 \text{ fish per hours fishing.}$$

Similarly the combined estimate for every other length group can be obtained, thus giving the whole length composition for that particular area. Where, as here, the same raising factors are used several times, the computational work can be reduced through eliminating the need for repeated division, by dividing the

raising factors to make their sum unity, i.e. here 10 + 8 = 18, so using raising factors equal to:

$$\frac{10}{18} \text{ and } \frac{8}{18}, \text{ i.e. } 0.556 \text{ and } 0.444, \text{ and}$$

the estimated density of 11 cm fish is 0.556 x 160 + 0.444 x 250 = 200 .This is then also done for every other area and the results can then be combined - most easily if all are in terms of the same catch per unit effort.

If no standard effort units can be used, then the length compositions estimated from the landings of each group of vessels will have to be expressed as percentages, and combined in this form.

Example 4.4.2.

A stock of sardines is fished by two groups of vessels in each of two different areas. In a certain year the number of samples taken and the resulting estimates of the number of fish caught in millions were as follows:

Area	Fleet	Samples	Length (cm)										
			18	19	20	21	22	23	24	25	26	27	28
Northern	A	50	1	8	15	28	30	27	20	13	7	4	2
	B	100	1	12	31	55	60	54	43	25	12	7	3
Southern	A	25	-	1	3	12	14	21	18	7	6	5	2
	B	20	-	2	4	13	13	23	18	10	5	2	1

Calculate the percentage length composition of the catches of each fleet in each area; hence estimate the percentage length composition of the stock in each area.

If the southern area is twice as big as the northern area, but echo surveys show that fish traces (assumed to be sardines) are 1.2 times as frequent in the northern area, estimate the percentage length composition of the whole stock (take weighting factors north: south of 1 x 1.2 : 2 x 1).

When there are appreciable differences in composition between the catches of different groups of vessels fishing in the same area, then clearly one group at least is selective, and catching an unrepresentative sample of the stock. Sometimes the nature of the selection can be guessed; for instance, if fish are being caught by trawl and gill net, then a difference between the catches, with the gill net catching relatively fewer very small and very big fish would be in agreement with

the expected selective action of the gill net. The composition of the stock would then be estimated from the catches of the presumably non-selective gear - in this example the trawl. If the selection can be measured reasonably accurately - as is possible for a fleet of gill nets if the mesh sizes in use are known - then the composition of the catches can be used to give an estimate of the composition of the stock, which can then be combined with the estimate from the landings by the other group of ships. In this latter step probably less weight should be given to the estimates from the selective gear than is indicated by the number of samples, because of the added uncertainty introduced by the selection factors used.

Example 4.4.3.

Herrings are caught in the southern North Sea by trawl and drift net. The length composition of the catches (in thousands), and the estimated proportion of each size retained by the drift-net (relative to the maximum retention) was as follows:

Length (cm)	21	22	23	24	25	26	27	28	29	30
Trawl numbers caught	30	320	1200	1400	1180	760	345	76	5	2
Drift " "	10	136	603	740	525	304	105	18	2	1
Drift, % retained	60	75	90	100	95	80	65	50	40	30

Estimate the percentage length composition of the stock, giving twice as much weight to the trawl estimates.

Even if a gear is not obviously selective it cannot be assumed that it is in fact non-selective. Though the meshes of the herring trawls in the example above are too small to let through any herring bigger than c. 15 cm, the catches could still be unrepresentative of the stock, e.g. the proportion of large herring could be underestimated because they tend to swim higher in the water and thus escape over the top of the trawl.

4.4.2. Age-length keys

Just as the age (or maturity, etc.) composition of the landings can be estimated by direct sampling, but is more effectively estimated by direct sampling for length, with the use of age-length (or maturity-length, etc.) keys based on relatively small samples, so may the age composition of the stock be best estimated by applying age-length keys to the estimated length composition. Because many of the selection processes by the fishing fleet are related directly to length, the use of

key enables their effect on age, etc to be readily determined. Thus, for the data on North Sea cod used earlier we can give the following age-length key, based on samples of the landings in the third quarter of this year.

	Number of fish of each age sampled								Total	Total
Length (cm)	1	2	3	4	5	6	7	8	Sampled	Landed (thousands)
30-	12	36							48	160
40-	1	73	3						77	488
50-	1	54	6						61	394
60-		13	25						38	205
70-			27	7					34	139
80-			8	12	7	3			30	75
90-				10	9	5	3		28	42
100-					4	2	7	3	15	15
110-						3	4	3	10	2

The known selective properties of the gear, and the market demand are such that no cod less than 30 cm are landed, and only a proportion of those under 40 cm which are caught are landed. Thus the landings can only give a reasonable representation of the stock for fish over 40 cm. The age-length key shows that therefore the landings may be representative of the stock of three-year old fish and older, but that the numbers of two-year old fish are slightly under-estimated, while the one-year old fish, most of which are under 40 cm, are very poorly represented.

If the selection in terms of length is known in quantitative terms, then the key can be used to estimate the selection in terms of age.

Example 4.4.4.

From the data given above estimate the numbers of fish of each age landed. If, of the fish that enter the trawl, only 40% of those between 30 and 40 cm, and 80% of those between 40 and 50 cm, are landed estimate the numbers of fish of each age, over 30 cm, which entered the trawl. By plotting the numbers of two-year old fish in each length group against length make a reasonable estimate of the number of two-year old fish less than 30 cm which entered the trawl. Hence estimate the percentage composition of the stock for fish two-year old and older.

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FB/T26 Manual of Sampling Methods for Fisheries Biology Oct. 1962

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FB/58/T1 A Manual of Field Methods in Fisheries Biology. (Preliminary draft) Sept. 1958

Drafts of "Manual of Laboratory Methods in Fisheries Biology" were issued as:

FB/58/T3 Part I Physical and Chemical Methods in Meteorology and Oceanography Jan. 1958

Part II Laboratory Methods on Plankton and Benthos Feb. 1958

Revised versions of these have been combined with the Field Methods manual to produce a "Manual of Methods in Fisheries Biology", the definitive text of which will be published in three language versions as FAO Man. Fish. Sci., (1) (in press).

A provisional English edition of FAO Man. Fish. Sci., (2) "Fisheries Science" was issued for FAO by CSIRO, Cronulla, Australia, in 1960.)